Abstract

In learning physics, students repeatedly encounter vectors (quantities which have a magnitude and a direction). They must be comfortable with vector notation and mathematics in order to solve many physics problems. Physics students are also presented with many types of representations, from graphs to field line diagrams. In solving physics problems students must be able to interpret this multitude of representations. I probed the intersection of students' vectors skills and their skills at interpreting visual representations. Specifically, my project sought to document the extent to which students are able to relate a vector representation of a physical situation to another representation of that same physical situation which does not use vectors. This ability was tested in various contexts throughout the introductory physics curriculum. I created pre-test questions which were administered to students at the University of Washington over three quarters. Responses to these questions were analyzed for correctness, common errors, and consistency. The specific results for each context are presented in the relevant sections of the thesis. However, in all contexts examined, students struggled to relate a vector representation to an alternate representation of a physical situation. Ideas for curriculum development to address this difficulty are presented at the end of this thesis.

An Investigation into Student Ability to Relate a Vector Representation to an Alternate Representation of a Physical Situation

Alexandra Lau

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Table of Contents

I. INTRODUCTION

Few physics professors would refute the importance of their students having a firm understanding of vectors. What may be less obvious is the extent to which we require our students to be able to transfer their knowledge of one representation of a physical situation (such as vectors) to another representation. This thesis focuses on student ability to relate a vector representation of a physical situation to an alternate representation of that same situation. This is examined in multiple contexts within the introductory curriculum.

By representation we mean a visual aid that models a phenomenon, process, or object from the real world. Examples of representations are an image of a physical situation or set-up; a graph; a field line diagram; and a time-lapse (strobe) diagram. While some representations are in three-dimensions, such as some computer simulations, the representations we consider here are in two-dimensions. In physics we use representations in many ways, e.g. to capture motion or to illustrate a physical process. We sometimes use representations to simplify a complicated structure; at other times we use representations (such as graphs) to compactly display a large amount of information about a situation.

We care about student ability to relate representations, particularly when one of them involves vectors, because students are routinely asked to do this in physics coursework. Consider the common task in an introductory mechanics class in which students are shown a strobe diagram of a car's motion (or a ball as it travels up a ramp), and then are asked to draw the acceleration (or velocity) versus time graph of the situation. In a strobe diagram, an object's location at equally spaced time intervals is shown. Students must be able to transfer this knowledge in both directions (from strobe diagram to motion graph and vice versa). Data collected this past summer at the University of Washington (UW) indicated that students may further have difficulty transferring their understanding of electric and magnetic field vectors to field line diagrams, which are commonly used in electromagnetism courses and textbooks. While the focus of this thesis is student understanding at the introductory level undergraduate courses, relating representations is a skill needed in advanced undergraduate courses as well. For example, students must relate the mathematical expression of a curl, gradient, or divergence to a vector diagram of the situation.

No matter the type of representation, ultimately students are supposed to use the representation to understand and explain something that goes on in the physical world. In order to gain this knowledge though, they must be able to interpret the representation and this includes relating it to other representations of the same situation.

The research presented in this thesis falls under the category of physics education research (PER). PER is the systematic study of the learning and teaching of physics. This subfield of physics research emerged over forty years ago. In fact physics was one of the first fields in science in which discipline-based education research (DBER) was conducted [1]. Discipline-based education researchers are physicists (or biologists, geologists, chemists, etc.) who are studying the learning and teaching of their subject. Over the last almost half-century, physics education researchers have devised systematic ways of studying learning in introductory physics courses; they have studied physics teacher preparation; and more recently have explored learning in upper division undergraduate, and even graduate level, physics courses [2, 3]. Even though extensive research has been conducted, there is much work still to be done at all levels of physics education [4-7].

Vectors are used throughout the physics curriculum, in lower and upper division courses. Students must be able to identify vector magnitude and direction; add and subtract vectors; and compute the dot and cross products. Of course, they must also be proficient in vector calculus for their advanced classes. Physics education researchers have studied student understanding of vectors in a non-physics context, and tests have been developed to measure students' basic vector understanding [8, 9]. Studies have also examined student understanding of vectors in a physics context and there have been comparison studies between the physics and non-physics contexts [9, 10]. Furthermore, researchers have studied student ability to relate graphs of a physical situation to the actual physical motion [11]. No work has examined, however, student ability to relate a representation of a situation that uses vectors to a representation that does not.

As with all the subfields of physics research, there are different approaches to, and lines of questions examined in, PER. This thesis falls under the group of PER that identifies student difficulties with particular topics and then works to correct them (applied PER). The physics education group (PEG) at the University of Washington views this work as a cyclical process, visualized in Figure 1. Because no one has studied my particular research question before, the majority of this thesis falls under the "Research" stage of the process shown in the Figure. It was simply not known how well students were able to transfer their understanding of a vector representation of a physical situation to another representation of that same situation. We examined students' ability to do this in the contexts of acceleration and of electric and magnetic fields. In order to study this properly, data had to be collected over multiple quarters (UW is on a quarter-system), and results from one quarter influenced the questions we asked the next quarter. Identifying the extent of student understanding and difficulties with a skill takes a lot of time, but is the

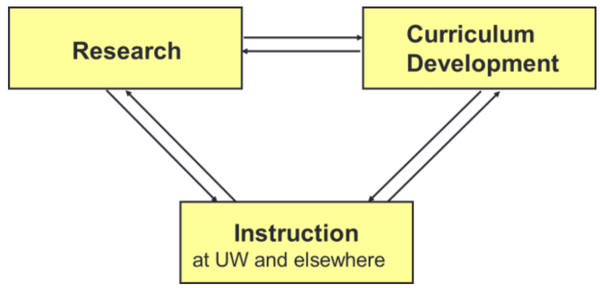


Figure 1: Iterative Research Cycle

essential first step in this strain of PER. Ideas for curriculum development will be discussed where appropriate, however.

For the selection of representations studied in this thesis, the following three broad questions were considered:

(1) *How well can students interpret the representation they are given?*

(2) *Are students consistent in their interpretation of multiple representations of the same situation?*

(3) *What conceptual and reasoning difficulties do students demonstrate with different types of representations? Are there common difficulties that emerge across representation type?*

II. GENERAL METHODOLOGY

Instructional Context:

At the UW, introductory physics is taught over a sequence of three quarters. The first quarter covers mechanics, the second quarter covers electromagnetism, and the third quarter covers waves, optics, and modern physics. The data presented in this thesis were collected from the first and second quarter classes. Students at the UW begin the study of a topic by watching an introductory video from the SmartPhysics program [12]. This online video presents the basics of the topic to be covered in class and contains questions that test student understanding of the material presented in the video. After completing the SmartPhysics video and activity, students attend lecture(s) on the topic. Courses include lecture, lab, and tutorial.

Following lecture(s) on certain topics, students attend a tutorial session related to the topic. These tutorials are small break-out sections of the larger lecture class in which students work in groups on material devised by the PEG to address known student difficulties. The materials used in this class come from *Tutorials in Introductory Physics* [13]. Before a tutorial session, students must complete a pre-test to which they will be referred during the tutorial session. One open-ended question on each midterm exam and a few multiple choice questions on the final exam are devoted to material coming specifically from the tutorials.

The PEG at UW creates the material used in the tutorials and continually assesses the effectiveness of the curriculum they create. Because the group is so heavily involved in the tutorials, there is plenty of opportunity to collect data from these classes for research purposes. The data presented in this thesis come from the tutorial classes.

Pre-tests for the tutorials:

As mentioned above, before each tutorial students must complete a pre-test which prepares them for the activity they will do in class. These pre-tests are administered through an online program. Students generally have fifteen minutes to complete the pre-test which includes a combination of multiple choice and open-ended questions. Some of the pre-tests include short animations for the students to watch. Each pre-test is multiple pages (students will answer the questions on the first page and then click a "next" button to move on to the next page). The pre-test can be administered such that students cannot go back to previous pages in the pre-test once they have moved on to the next page. Pre-tests are graded on completion, not correctness.

Data Analysis:

Data presented in this thesis comes from student responses to multiple choice questions on tutorial pre-tests. Explanations of their reasoning were also collected. We looked at aggregate performance of the whole class as well as individual student performance. Responses were examined for correctness, but common incorrect responses were also analyzed.

III. EXAMINATION of STUDENT ABILITY to RELATE REPRESENTATIONS

In this chapter the tasks administered to the introductory classes at the University of Washington will be presented and discussed. The tasks fall into two main categories that describe the types of representations the students were comparing. The tasks, which required students to relate the vector representation of a situation to a graphical representation, will be presented first. Next, student ability to relate a vector representation to a field line representation will be examined.

A. Vector to Graphical Representation

Task Description:

One of the first times physics students must relate a vector representation to a graphical representation is soon after they learn about vectors, in their study of velocity and acceleration.

In the pre-test to the Acceleration in One-Dimension (A1D) tutorial, students see an animation of a ball rolling up and then down an inclined plane. The animation runs only once, but students are also given a strobe diagram of the ball's motion [Appendix 1]. Students are asked to choose an arrow (from eight options given) that represents the direction of the ball's acceleration when it was (1) at a specific point and ascending; (2) at the top of the incline; and (3) at a specific point and descending. This set of questions, we will call them the Vector Questions, were meant to measure student ability to correctly identify the vector representation of the ball's acceleration at various locations along its path of motion.

Students can answer this question using forces or the definition of acceleration (change in velocity divided by change in time). When students took the pre-test, they had not yet covered forces in lecture, although many of the students had seen forces in their high school physics course. At this point in instruction, all students should have been able to answer this question by noting that the ball was slowing down as it ascended the ramp, meaning its change in velocity over change in time was directed opposite its velocity. On descent, students should have recognized the ball was speeding up (its velocity was getting bigger down the ramp) which made its change in velocity divided by change in time directed down the ramp. By the spacing of the balls in the strobe diagram, students could figure out that the magnitude of the acceleration was constant (although that was not strictly required to answer the question). Using this reasoning, students should get the correct answer that the acceleration is directed down the ramp and has constant magnitude during the whole motion. (This is represented by letter choice "e" for all three points).

In the Fall 2014 term, we added a question following the Vector Questions. Students were asked to choose the velocity vs. time graph that represents the ball's motion as it goes up and then down the ramp. Students were told to assume that the positive direction was up the ramp. This Graph Question measures student ability to relate the ball's motion to a graphical representation of that motion. To answer this question correctly, students must understand that the slope of a velocity vs. time graph is acceleration. The ball has constant acceleration in the negative direction throughout its motion, so the correct graph choice is "A," a line with a constant, negative slope (see Appendix 1).

By comparing performance on the Vector Questions and the Graph Question, we can see if students are consistent in their answer choices. In other words, to what extent are they consistent in their choice of vector diagram and their choice of velocity vs. time graph?

Fall Quarter Results:

Table 1: A1D Results from Fall Quarter
***Percentages rounded to nearest 5%.**

	Percent Correct (n=564)
Point 3	50%
Point 6	30%
Point 10	65%
All Points	30%
Graph	60%
All Points & Graph	25%

Overall Pretest Results: Acceleration Vectors

Table 1 presents the results from the administration of the pre-test in the autumn quarter. Of the 564 students who took the pre-test, 50% correctly identified the direction of the acceleration when the ball was at point 3 (ascending) and a higher percent, 65%, of students correctly identified the direction of acceleration when the ball was at point 10 (descending). Students had the most trouble at the turn-around point, point 6. Only 30% of students correctly identified the direction of the acceleration at that location. Although the percent correct varies for each point, when looking at responses for all three points together, only 30% of students chose the correct arrow for each point.

Table Two presents the most common incorrect answers to the Vector Questions. Ten percent of students chose “b,” the arrow pointing straight down, for each of the three points. Many of these students explained that the acceleration of the ball was due to gravity, which points straight down. These students were not thinking about the change in velocity. They seem to be taking a force approach to answering this question (and applying it incorrectly). While students had not covered forces in class at the time of this pre-test, many of the students had taken high school physics and would have been familiar with the concept of force.

Ten percent of the students correctly identified the direction of the acceleration at points 3 and 10, but incorrectly believed that the acceleration was zero at the top of the ramp (point 6). Fifteen percent of students said the acceleration was directed up the ramp at point 3, was zero at point 6, and was directed down the ramp at point 10. This is incorrect because the acceleration is not zero at the top of the ramp (although the velocity is). It is also incorrect because on the ascent, the ball’s acceleration is in the opposite direction of the ball’s velocity (acceleration is down the ramp). This common incorrect line of answers is consistent with students confusing velocity with acceleration.

Table 2: Most common errors on Vector Questions (Fall 2014)

***Percentages rounded to nearest 5%.**

Incorrect Answer Choices for Direction of Acceleration at points 3; 6; and 10	**Percent of Students who made the Error**
Straight down at all 3 pts	10%
Up the ramp (pt 3); zero (pt 6); down the ramp (pt 10)	15%
Down the ramp (pt 3); zero (pt 6); down the ramp (pt 10)	10%

Overall Pretest Results: velocity vs time graph

On the Graph Question, 60% of students chose the correct answer. Since we were asking students for the velocity vs. time graph, this required students to recognize the acceleration as the slope of the graph they chose. The most common incorrect choice for this question, made by 20% of students, was choice "B" which has a line with a negative slope that intersects the t-axis and then connects to a line with a positive slope.

Consistency between vectors and graph

Comparing the results for the Vector Questions and the Graph Question lets one examine how well students were able to relate the vector representation to a non-vector representation of the same physical situation. Twenty-five percent of students chose the correct arrow for all three points asked about along the ramp and chose the correct velocity versus time graph. Of the 165 students who chose the correct arrow for all three points, 133 of them (80%) chose the correct velocity vs. time graph as well. This tells us that if a student knows how to form the vector representation (correctly chose the acceleration arrow at *all* three points) then they probably will be able to relate this to their v vs. t graph choice.

However, many students who chose an incorrect arrow for at least one of the points asked about in the first set of questions are not consistent when they then make their v vs. t graph choice. Consider the students who say that the acceleration is zero at point 6 (the top of the ramp). Of the 216 students who make this mistake, 119 (or 55%) then make the correct choice for the v vs. t graph. The correct answer (A) is a line with a constant negative slope. These students fail to connect their choice of a zero vector at point 6 with their choice of a graph which indicates that the acceleration is never zero. Again, this may correspond to confusion between acceleration and velocity.

Modification to task (Winter 2015)

In the Winter 2015 term, there were two sections of the Introductory Mechanics class and we administered a different version of the pre-test to each section of the class. Both versions of the pre-test started with the Vector Questions that were asked of students in the Fall term. This set of questions was followed by the Graph Question. On one version of the pre-test, we asked students to select the velocity vs. time graph that represented the ball's motion. We changed the answer choices slightly from the Fall version based on what we observed students responding in the Fall administration of the pre-test (See Appendix 1, Modifications Version A). On the other version of the pre-test, students were asked to choose the acceleration vs. time graph that represented the ball's motion (See Appendix 1, Modifications Version B). We ran this question to see how well students were able to relate a vector representation of a situation to a different type of graphical representation (acceleration vs. time) than asked about previously. Furthermore, when choosing a velocity vs. time graph, students must first recognize that the slope of the graph represents the acceleration. By asking for an acceleration vs. time graph we eliminated this middle step and therefore anticipated that it would be easier to look for consistency in students' responses to the Vector Questions and Graph Question. (To correctly answer the acceleration vs. time graph question students only needed to recognize that the ball had constant, negative acceleration throughout its motion, and this is represented by a horizontal line below the t-axis on an acceleration vs. time graph).

Overall Pretest Results: Acceleration Vectors

The results from Winter quarter are presented in Table 3. The same Vector Questions were asked on both versions of the acceleration in 1-D (A1D) pre-test. A total of 163 students took the pre-test. About 45% of the students correctly identified the direction of the acceleration of the ball at point 3 (on its ascent). Thirty-five percent of students chose the correct direction at point 6 (when the ball is at the top of the ramp). Sixty percent of students chose the correct direction at point 10 (the ball is descending). Overall, 35% of students chose the correct direction of the acceleration at all three points. These results for the

Vector Questions are consistent with results from Fall Quarter when the identical questions were asked (see Table 1).

Table 3: A1D Vector Questions Results from Winter Quarter

***Percentages rounded to nearest 5%**

	Percent Correct (n=163)
Point 3	45%
Point 6	35%
Point 10	60%
All Points	35%

Table 4 reports the most common incorrect sets of answers for the three Vector Questions. The results are very similar to those from Fall Quarter. Twenty percent of students chose the vector pointing straight down (answer choice "b") as the direction of the acceleration for all three points. The next most common mistake, made by 15% of students was to say acceleration was directed up the ramp (choice "f") at point 3, was zero (choice "g") at point 6, and was directed down the ramp (choice "e") at point 10. Another common error was choosing the direction of the acceleration to be down the ramp (choice "e") at points 3 and 10, but for the acceleration to be 0 at point 6. This line of answers was chosen by 10% of students.

Table 4: Most Common Errors on Vector Questions (Winter Quarter)

***Percentages rounded to nearest 5%**

Incorrect Answer Choices for Direction of Acceleration at points 3; 6; and 10	Percent of Students who made the Error
Straight down at all 3 pts	20%
Up the ramp (pt 3); zero (pt 6); down the ramp (pt 10)	15%
Down the ramp (pt 3); zero (pt 6); down the ramp (pt 10)	10%

Overall Pretest Results: velocity vs time graph and acceleration vs time graph

One-hundred twenty-two students took the version of the A1-D pre-test whose Graph Question asked for the velocity vs. time graph of the ball's motion. Of these students, 65% chose the correct graph (see Table 5). This result is consistent with the percent of students who chose the correct velocity vs. time graph in Fall quarter. Forty-one students took the other version of the pre-test which asked for the acceleration vs. time graph of the ball's motion. Of these students, about 45% chose the correct graph.

Table 5: A1D Graph Question Results from Winter Quarter

***Percentages rounded to nearest 5%**

	Percent Correct
v* vs. *t (n=122)	65%
a* vs. *t (n=41)	45%

Table 6: Common Incorrect Responses to the Graph Question (Winter Quarter)

***Percentages rounded to nearest 5%**

	Percent of Students who Made the Error
v* vs. *t (n=122)	
Incorrect Answer: B	20%
a* vs. *t (n=41)	
Incorrect Answer: A	20%
Incorrect Answer :F	15%

The most common incorrect answer on the velocity vs. time graph question was choice "B," a line with a negative slope that decreases to zero then switches to a line with positive slope. Twenty percent of the students who answered this version of the pre-test put this answer (see Table 6).

The most common error on the acceleration vs. time graph question, made by 20% of the students who answered that version of the pre-test, was choice "A". Choice "A" is a straight line with a negative slope. (That would be the correct answer choice for a *velocity vs. time* graph). The other common mistake on the acceleration vs. time graph question was to choose "F," a line with a negative slope then switching to a line with positive slope. Fifteen percent of the students who answered this version of the pre-test made this error.

Consistency between vectors and graph

To look for consistency in students' answers, we first consider the students who took the version of the pre-test with the velocity vs. time question as the Graph Question. Of the 122 students who took this version, 25% of them chose the correct direction at all three points along the ramp, and chose the correct velocity vs. time graph. These students were consistent in their answers for the Vector Questions and the Graph Question.

Of the 40 students who picked the correct vectors at all the points, 75% of them chose the correct graph as well. In other words, the majority of the students who correctly identified the direction of the acceleration of the ball at all three points also chose the correct velocity vs. time graph depicting the ball's motion.

Sixty-five percent of the students chose the correct velocity vs. time graph, and of these students, 65% chose the same arrow at all three points for the Vector Questions. These students were consistent in recognizing the acceleration was constant in both the vector and graphical representations.

Forty students said there is zero acceleration at point 6 (the top of the ramp). Fifty-five percent of these students then chose the correct velocity vs. time graph, a line with a constant negative slope (acceleration). One possible interpretation of this is that the students were inconsistent because they say acceleration is zero at one point in the Vector Questions and then non-zero on the graph question. However, this assumes they recognize the slope is the acceleration. (Also, if students were thinking the first set of questions was asking about velocity, then their graph choice is consistent). It is hard to say which is happening; there were likely some students who were confusing velocity and acceleration.

Now we will consider the students who took the version of the pre-test with the acceleration vs. time question as the Graph Question. (The number of students who took this version of the pre-test, forty-one, was small). Twenty-five percent of these students had the correct answer on all the Vector Questions and on the Graph Question. These students were consistent in their answer choices. This percentage is also very similar to the same statistic calculated for the class which took the velocity vs. time version of the pre-test.

Of the thirteen students who chose all the correct vectors, 75% of them chose the correct graph as well. Again, this is similar to the same statistic calculated for the velocity vs. time class.

Overall, 45% of the students who took the acceleration vs. time graph version of the pre-test chose the correct graph. Of these students, 90% had picked the same direction at all three points for the Vector Questions. (Note that this does not mean they picked the *correct* same direction). These students recognized that the ball had constant acceleration in both representations they were asked to consider.

Commentary on Results:

Our main research question is, "To what extent can students relate a vector representation of a situation to another representation of the situation which does not use vectors, and what are some common errors?" Table 7 summarizes the results that help us answer this question.

Table 7: Summary of A1D Results.

***Percentages rounded to nearest 5%.**

	Fall Quarter	**Winter Quarter, v vs. t version**	**Winter Quarter, a vs. t version**
Correct on all Vector Questions and the Graph Question	**25%** of class	**25%** of class	**25%** of class
	80% of those who got all the Vector Questions correct	**75%** of those who got all the Vector Questions correct	**75%** of those who got all the Vector Questions correct
Same arrow choice on all 3 Vector Questions	**55%** of those who chose the correct graph	**65%** of those who chose the correct graph	**90%** of those who chose the correct graph

Perhaps the simplest way to answer this question is to say that only the students who are able to correctly answer the Vector Questions and the Graph Question are the ones who can relate representations. After all, these students are able to create a correct vector representation of the ball's acceleration and identify the correct graphical representation of the ball's motion. They are consistent between the questions, demonstrating that they relate the information presented in either representational format. By this measure, only one-quarter of students correctly related the representations. Recall, students had seen this material in their textbook and lecture prior to taking our pre-test. This indicates that students have significant difficulty relating a vector representation of acceleration to a graphical one (regardless of if this graph is acceleration or velocity vs. time).

Another way to consider our research question is to examine the students who get all questions correct as a percent of the students who get all the Vector Questions correct. Between 75 and 80% of the students who chose the correct acceleration vector at all three points also answered the Graph Question correctly. The majority of students who correctly created the vector representation were then able to choose the correct graph. This required them to relate the information in the vector representation (that the acceleration was constant) to the graphical representation. However, these results also tell us that 20-25% of the students who pick the correct acceleration direction at all three points then fail to choose the correct graph of the ball's motion. These students are not consistent in their answers for the two types of representations and this indicates an inability to relate the representations. (We believe that if a student can relate representations, their answers are consistent for the different representations). The results imply that even if students can identify the correct vector representation of an object's acceleration, up to one-quarter of them are not able to identify the graphical representation of the object's motion.

Additionally, we can consider the students who chose the same arrow for all three points along the ramp as a percent of those who chose the correct graph. Fifty-five to 65% of the students who chose the correct velocity vs. time graph chose the same (although not necessarily the correct) acceleration direction at all three points. (The 10% difference in performance may be due to the smaller sample size in Winter Quarter, and the more challenging distractors used in the Winter version. As discussed previously, we modified the answer choices (the distractors) in the Winter version to align with common incorrect answers we saw students making in the Fall. We expected students to perform worse on the Winter version because the new distractors should have made it more challenging). The percent of students who correctly answered the Vector and Graph Questions turned out to be the same for the Fall version and Winter version (with velocity vs. time graph). However, the slight difference (10%) discussed above is still reasonable because the questions (their answer choices) were not exactly the same.

Ninety percent of students who chose the correct acceleration vs. time graph chose the same acceleration direction at all three points. This is much higher than what we see for the velocity vs. time versions of the pre-test, but keep in mind the sample size for this acceleration vs. time version of the pre-test is <50 students. (Indeed, it is important to run this question again to increase our sample). Also note, the results show that 45% of students get the acceleration vs. time graph correct while 65% get the velocity vs. time graph correct. This result is unexpected because we thought the acceleration vs. time graph would be easier as students do not have to know what the slope represents in the graph. It may be that students are less familiar with the acceleration vs. time graph, so we saw such a high percent (90%) of students who chose the correct acceleration vs. time graph chose the same acceleration direction at all three points because we are selecting for students with more background in kinematics.

The percentages tell us that just because a student can correctly choose a graph, does not mean they can relate this to another representation type. The graph appears to be the easier (or perhaps more familiar, memorized) representation (around 30% of students get all the vectors correct while 45-65% of students get the graph question correct) but our results indicate that choosing the correct graph does not imply understanding that acceleration is constant at all three points along the path. The major instructional implication to come from this is that if an instructor wants to test knowledge of acceleration, using a vector representation question will be more discriminating than a graphical representation question. (Although using an acceleration vs. time graph may be fine. Our acceleration vs. time graph results indicate that the acceleration vs time graph is a better indicator that students will get the vectors correct than the velocity vs time graph).

B. Vector to Field Lines and Vice Versa

We now consider tasks that required students to relate a vector representation to a field line representation, and tasks that required the opposite (for them to relate a field line representation to a vector representation). The ability to relate these types of representations is necessary in an electromagnetism class. The results presented below are from the electromagnetism course that students take in the second quarter of the introductory physics sequence at UW. The first example is from electric fields, and the second is from magnetic fields.

1. Electric Potential Difference Pre-test

Students work through the Electric Potential Difference tutorial early in the quarter, and this tutorial is preceded by a pre-test. In the Fall of 2014, two versions of this pre-test were run. On each version we included two questions to probe student ability to relate a vector representation to a field line representation. (Pre-test questions are included in Appendix 2). The tutorial and its pre-test come after students have had lecture instruction on electric field vectors, electric field lines, and electric potential differences.

Task Description:

The first question on both versions of the pre-test was identical. I call this question the Vector Question. Students were shown two oppositely charged point charges separated by a fixed distance. They were also shown a number of arrows pointing in a variety of directions. Students were asked to choose the arrow that points in the direction of the electric field vector at four marked locations around the point charges. Three of these locations lay along the line through the two charges. The fourth location was at a distance perpendicular to the line the charges lay along. There was also an option that the field is zero and an option of "none of the above choices."

One way students could answer this question correctly was to understand that field lines are directed away from positive charges and towards negative charges. Students could also have a sense (although they needn't have memorized the exact electric field equation) that the field strength increases with decreasing distance from a point charge. Furthermore, they could add the contribution to the field from each point charge (do simple vector addition). Students who successfully understood the above would get the correct answers: the field is directed straight down at point 1 and point 3, and straight up at point 2 and point 4. (The correct answer sequence, for points 1 to 4, was GAGA).

On the next question, students saw the same charge configuration, but now with field lines in place. I will call this the Field Line Question. The two versions differed in the field lines shown. Version One had field lines running through all the marked points, while Version Two's field lines ran through none of the points. Students (in both versions) were asked to do the exact same thing they had done in question one: choose the arrow (same options given on both versions of the test) that best represents the direction of the electric field vector at the marked points. Again there was an option that the field was zero and a "none of the above choices" option. Students could not go back to the Vector Question to change their answers once they saw the Field Line Question.

In previous administrations of older versions of this pre-test we noticed that some students said the field was zero in-between the point charges. We also wondered if students thought the field ended (was zero) outside the drawn field lines and if the field was zero at points not on a field line. This was our motivation for running Version Two of the pre-test in the fall. We wanted to see how responses would change if the field lines did not run directly through the points we asked about.

The answer to both versions of the Field Line Questions is the same, and furthermore is the same as the answer to the Vector Questions. Students needed to recognize that field lines point in the direction of the field. In fact, they are built by drawing (small) field vectors at each point and connecting them. At each point, the direction of the electric field vector is tangent to the field line at that point. Therefore, the correct answers to both versions of the Field Line Questions are an arrow pointing straight down at points 1 and 3, and an arrow pointing straight up at points 2 and 4. (In terms of the letter options this corresponds to GAGA).

Fall Quarter Results:

The question without field lines present, the Vector Question, was present on both versions of the EPD pre-test administered in the fall. A total of 375 students answered this question, and 40% of them chose the correct set of arrows indicating the direction of the electric field at four marked points (see Table 8). Two-hundred and ten of these students took Version One of the field line question which followed the Vector Question. The field lines ran through the four points, and 65% of the 210 students chose the correct set of arrows indicating the direction of the electric field at the points. The other 165 students took Version Two of the field line question, which also followed the Vector Question. In this version the field lines did not run through the marked points, and 45% of the students chose the set of arrows which correctly indicated the direction of the field at all four points.

Table 8: *Correct* Performance on EPD from Fall Quarter

***Percentages rounded to nearest 5%**

Vector Question (n = 375)	**Field Line Question, Version One- Lines Through Points** (n=210)	**Field Line Question, Version Two- Lines *Not* Through Points** (n=165)
40%	65%	45%

The most common incorrect responses on the Vector Question and Field Line Question are shown in Table 9. One-quarter of all the students who took the EPD pre-test (either version) said on the Vector Question that the field was zero at point 2, which lay in between the two point charges. On the Field Line Question, when a field line ran through this point (Version One), 5% of the students said the field was zero at point 2. For Version Two, 15% of the students said the field was zero at point 2.

The other common mistake was to choose the correct direction of the field at points 1-3 (all lay along the axis that runs through the point charges), but to choose an arrow that curved towards the negative point charge at point 4. (This answer choice follows the shape of the field line around point 4, but the correct direction is the tangent to the field line at point 4). On the Vector Question, 15% of students made this error; on Version One of the Field Line Question 15% of students made this error; and on Version Two of the Field Line Question, 20% of students made the error.

On Version Two of the Field Line Question, point 4 is outside all the drawn field lines. One of the reasons we designed the question this way was because we wanted to see if students thought the field ended (was zero) outside the drawn field lines. Only 3% of the students who took this version said the field was zero at point 4.

Table 9: Common Incorrect Responses on Fall Quarter EPD

***Percentages rounded to nearest 5%**

	Vector Question (n = 375)	**Field Line Question, Version One- Lines Through Points** (n=210)	**Field Line Question, Version Two- Lines *Not* Through Points** (n=165)
Zero at point 2	25%	5%	15%
Correct at points 1-3, but chose curved arrow at point 4	15%	15%	20%

Modification to task (Winter 2015):

In the Winter Quarter there were three sections of the electromagnetism class and we ran three modified versions of the EPD pre-test (see Appendix 2). The Vector Question was unchanged, but the images used in the Field Line Questions (both versions) were modified slightly to improve the accuracy of the representations. Students were asked about the direction of the electric field vector at the same four points as in the Fall versions of the pre-test. I label the three versions we ran in the winter as Version Three, Version Four, and Version Five, so as not to confuse them with the versions (One and Two) we ran in the Fall.

We purposefully designed the order of the questions in Versions Three and Four to be opposite of that run in the Fall (i.e. in the Winter the Field Line Question came before the Vector Question). We did this to eliminate order effects. (Running the questions in opposite orders in the different versions allowed us to see the true effect of the question type and not just the effect of the order the questions appear in). Version Three had the Field Line Question with the lines running through the four points we asked about. This question was followed by the Vector Question. Version Four had the Field Line Question with the lines that do not run through the four points followed by the Vector Question.

Version Five of the pre-test had the Vector Question followed by the Field Line Question where the lines do not run through the points we ask about. This version was very similar to Version Two of the pre-test except that the image used for the Field Line Question was modified slightly.

Winter Quarter Results:

All three versions of the EPD pre-test administered in the Winter contained the same Vector Question. Out of the 390 students who answered the Vector Question, 45% chose the correct set of arrows indicating the direction of the electric field at the four marked points (see Table 10).

On Version 3 of the pre-test, the Field Line Question had lines that ran through the points we asked about. Sixty percent of the students who took this version chose the correct set of arrows for the direction of the electric field at the four points. Versions 4 and 5 of the pre-test contained the Field Line Question where the lines did not run through the points we asked about. Of the students who took this version of the pre-test, 40% chose the correct set of arrows indicating the direction of the electric field at the four marked points. These results are similar to those from the Fall administration (see Table 8).

Table 10: *Correct* Performance on EPD from Winter Quarter

***Percentages rounded to nearest 5%**

Vector Question (n = 390)	**Field Line Question (Version 3) Lines Through Points** (n=103)	**Field Line Question (Version 4 & 5) Lines *Not* Through Points** (n=287)
45%	60%	40%

Table 11 shows the common incorrect responses on the EPD questions. On the Vector Question 15% of student said the field was zero at point 2 (the point in between the two charges). Fifteen percent of students chose the correct direction of the field at points 1 – 3 but chose an arrow that curved towards the negative charge for point 4.

On the Field Line Question where the lines go through the marked points, 10% of students said the field was zero at point 2 and 10% of students chose the correct direction of the field at points 1-3 but chose a curved arrow at point 4.

On the Field Line Question where the lines do not go through the marked points, 15% of students said the field was zero at point 2. Fifteen percent of students chose the correct field direction for points 1-3 but chose a curved arrow for the direction of the field at point 4. Point 4 lay outside all of the drawn field lines. Eight percent of students said the field was zero at point 4.

Table 11: Common Incorrect Responses on Winter Quarter EPD

***Percentages rounded to nearest 5%**

	Vector Question (n = 390)	**Field Line Question, (Version 3) Lines Through Points** (n=103)	**Field Line Question, (Version 4 &5) Lines *Not* Through Points** (n=287)
Zero at point 2	15%	10%	15%
Correct at points 1-3, but chose curved arrow at point 4	15%	10%	15%

Consistency between Vector and Field Lines Questions (Both Quarters):

On each version of the EPD pre-test we ran students had a Vector Question and a Field Line Question. On both the Vector Question and the Field Line Question students were asked to do the same thing: Choose the arrow from among the options shown that best represents the direction of the electric field vector at the four marked points. The four points we asked about stayed the same for each version of the pre-test. The difference between the pre-test versions lay in (1) the order in which questions were asked and (2) the images used for the Field Line Question.

To answer our overarching research question, “Are students able to relate a vector representation of a situation to a representation of the same situation which does not use vectors?” we must examine if students are consistent in their responses on the Field Line Question and the Vector Question. Again, the only difference between these two question types is the representation presented to the students. The answer to both questions is the same. For both of the question types, we counted a response as correct if the student chose the correct direction of the electric field at *all* four points asked about. If the student chose an incorrect direction at even just one of the four points, the answer for that question was counted as incorrect.

Version One of the pre-test was administered in the Fall and it had the Vector Question followed by the Field Line Question with the lines going through the points. Of the 210 students who took this version, 35% of them answered both questions correctly (see Table 12). These students were consistent in their answer choices. Twenty-five percent of the students answered both questions incorrectly, although only 5% of students chose the exact same incorrect response for both questions. Ten percent of the students answered the first question, the Vector Question, correctly and then answered the Field Line Question incorrectly. However, 30% of students answered the Vector Question incorrectly and then answered the Field Line Question correctly. We also saw 20% of students say the field was zero at point 2 (the point in the middle between the two charges) on the Vector Question and then chose the correct direction at that point on the Field Line Question.

The corresponding percentages detailing consistency for Versions Two, Three, Four, and Five are reported in Table 12. Note that the first row of the table describes the order in which the questions were asked on each version of the pre-test. For example, in the first row, fourth column of the table it says “V5 Vector, then FL not through points." This means that on Version Five of the pre-test the Vector Question was asked first and was then followed by the Field Line Question where the lines do not flow through the points.

Table 12: Consistency on the Vector and Field Line (FL) Questions, Version 1 & 2 (Fall Quarter), Version 3-5 (Winter Quarter) of EPD

***Percentages rounded to nearest 5%**

	V1 Vector, then FL through pts (n=210)	V2 Vector, then FL *not* through pts (n=165)	V5 Vector, then FL *not* through pts (n=114)	V3 FL through pts, then vector (n=103)	V4 FL *not* through pts, then vector (n=173)
Both Vector and Field Line Questions **Correct**	35%	35%	30%	50%	40%
Both Vector and Field Line Questions **Incorrect**	25%	50%	60%	35%	45%
Correct on Vector Question, **Incorrect** on Field Line Question	10%	5%	5%	5%	10%
Incorrect on Vector Question, **Correct** on Field Line Question	30%	10%	10%	10%	5%
Put Zero at Point 2 on Vector Question, and Correct Direction at Point 2 on Field Question	20%	10%	10%	5%	5%
Put Same Incorrect Answer for Both Questions	5%	20%	25%	15%	25%

Commentary on Results:

The data in Table 12 show that regardless of the order in which the questions were presented and of which Field Line Question they had, at most half of the class was able to answer both the Vector Question and the Field Line

Question correctly. (And in some cases the percent of students who answered both questions correctly was almost 20% lower than this). This is surprising because the answers to both of the questions on the pre-test were the exact same. The only difference is that the figure used in the Vector Question had no field lines while the figure used in the Field Line Question had field lines drawn. Students were not seeing this material for the first time when they took the pre-test. On the contrary, they had been introduced to the information multiple times through pre-lecture work and lecture class.

The results also suggest that if students are presented with the field line question prior to the vector question, they are more likely to get both questions right than if they are presented with the questions in the opposite order (see row two of Table 12). This may imply that field lines help students identify the direction of the electric field. They at least do not seem to hurt student understanding of the field direction. As shown in Tables 8 and 10, the percent of students correctly choosing the direction of the electric field at the four marked points is comparable or better when there are field lines to when there are no field lines.

Students were consistent on the pre-test if they either answered both questions correctly or chose the same incorrect answer for both questions. On the versions of the pre-test where the Field Line Question preceded the Vector Question, a larger percent of students were consistent in their answers than when the questions were presented in the opposite order. In other words, students were more likely to put the same field directions they chose on the Field Line Question for the Vector Question answers as well, than the students who saw the Vector Question first.

When the Field Line Question had an image with the field lines not running through the marked points, a larger percent of students put the same incorrect answer for both questions than when the image showed the lines running through the points. This suggests that students interpret the field line diagram with lines not running through the marked points similarly to how they interpret the diagram without field lines. This adds nuance to the results discussed above, which indicate that field lines help students identify the direction of the electric field vector. It appears that the type of field line diagram presented to the students matters. Consider Version One of the pre-test where students first answered the Vector Question and then answered the Field Line Question with the lines running through the marked points. As Table 12 shows, 30% of the students answered the Vector Question incorrectly and then the Field Line Question correctly. This indicates that the field line representation (with the lines running through the marked points) helped students choose the direction of the electric field at the points. In contrast, on Versions Two and Five where the Vector Question was followed by the Field Line Question with the lines not running through the marked points, only 10% of students went from incorrect to correct in their responses to the Vector Question and Field Line Question, respectively. When the field lines do not run through the points we ask about, they may not be as useful to students. We need to run more administrations of these versions of the pre-test to see if these results are significant.

2. Magnetic Interactions Pre-Test

The magnetic interactions (MGI) pre-test is taken by students during the second half of their electromagnetism class. In the Fall of 2014, we administered two versions of this pre-test to the electromagnetism classes. On each version, we asked two questions that probe student ability to relate a vector representation to a field line representation, in the context of magnetic fields. The pre-tests are included in Appendix Three.

Task Description:

For one of the questions, students are shown a bar magnet with north and south poles marked. They are asked to select the arrow (from options shown to them) that most accurately indicates the direction of the magnetic field vector due to the bar magnet at each of four marked points. I will call this question the Magnetic Vector Question. To answer this question correctly, students must understand that the magnetic field (and thus the magnetic field vector) points out from the north pole and into the south pole, and the field ‘circulates.’ If students understand this, they should say that the field points straight up at points 1-3 and straight down at point 4. (This corresponds to the letter options AAAG).

The other question we asked showed a bar magnet oriented the same way as in the Magnetic Vector Question, but now field lines were drawn in. Students were asked the same question as for the Magnetic Vector Question, and the same four points were marked. There were two versions of this "Magnetic Field Line Question." On one version, the field lines ran through the marked points. On the other version the lines did not run through the marked points. The answer to both versions of the question was the same as the answer to the Magnetic Vector Question. Students had to recognize that field lines point in the direction of the field and the field at a point is tangent to the magnetic field line at that location. They had to know the field comes out of the north pole of the magnet and flows into the south pole of the magnet.

What I will call Version One of the MGI pre-test had the Magnetic Vector Question followed by the Magnetic Field Line Question with the lines not running through the marked points. Version Two of the MGI pre-test had the Magnetic Field Line Question with the lines running through the marked points followed by the Magnetic Vector Question.

Results:

A total of 369 students answered the Magnetic Vector Question. Of these students, 25% answered the question correctly (see Table 13). For a response to be counted as correct, a student had to correctly identify the direction of the magnetic field at *all* four points. Two hundred and nine students took Version One of the pre-test which contained the Magnetic Field Line Question where the lines did not run through the marked points. Of these 209 students, 40% correctly answered the Magnetic Field Line Question. The other 160 students took Version

Two of the pre-test which contained the Magnetic Field Line Question with the lines going through the marked points. Forty-five percent of these students correctly answered the Magnetic Field Line Question.

Table 13: *Correct* Performance on MGI

***Percentages rounded to nearest 5%**

Vector Question (n = 369)	**Field Line Question, Version One- Lines *Not* Through Points** (n=209)	**Field Line Question, Version Two- Lines Through Points** (n=160)
25%	40%	45%

Students' most common incorrect answers on the MGI pre-test are shown in Table 14. A number of students said the field was zero at the point inside the magnet (point 2). Forty percent of the students who answered the Magnetic Vector Question made this error. Twenty-five percent of students who answered the Magnetic Field Line Question where the lines did not run through the points made this mistake. On the Magnetic Field Line Question with the lines running through the marked points, 35% of students said the field was zero at the point inside the magnet. It is surprising that this percent is higher than that for the version of the Magnetic Field Line Question where the lines did not run through the points, but this could be an order effect.

Another common error was choosing an arrow curved towards the south pole of the magnet for the direction of the field at point 4. (This corresponds to answer choice

"F"). This is incorrect because the direction should be straight down (tangent to the field line for the Magnet Field Line Question). On the Magnetic Vector Question, 15% of students made the error of choosing a curved arrow at point 4. On the Magnetic Field Line Question where the lines do not run through the points, 20% of the students made this error. On the Magnetic Field Line Question where the lines run through the points, 15% of students made this error.

Table 14: Common Incorrect Responses on MGI

***Percentages rounded to nearest 5%**

	Vector Question (n = 369)	**Field Line Question, Version One- Lines *Not* Through Points** (n=209)	**Field Line Question, Version Two- Lines Through Points** (n=160)
Zero at point 2	40%	25%	35%
Curved arrow choice at point 4	15%	20%	15%

Consistency:

Table 15: Consistency on the Magnetic Vector and Magnetic Field Line (FL) Questions for both Versions 1 and 2 of MGI *Percentages rounded to nearest 5%

	V1 Vector, then FL *not* through pts (n=209)	V2 FL through pts, then Vector (n=160)
Both Vector and Field Line Questions **Correct**	15%	35%
Both Vector and Field Line Questions **Incorrect**	60%	55%
Correct on Vector Question, **Incorrect** on Field Line Question	0%	0%
Incorrect on Vector Question, **Correct** on Field Line Question	25%	10%
Put Same Incorrect Answer for Both Questions	15%	30%

To examine student ability to relate the representation types, we looked for consistency of responses. Table 15 reports these consistencies (and inconsistencies). On Version One of the pre-test, 15% of students answered both the Magnetic Vector Question and Magnetic Field Line Question correctly. A much larger percent, 60%, of the students answered both questions incorrectly. However, only 15% of the students put the same incorrect answer for both questions. Almost no student answered the Magnetic Vector Question correctly and then incorrectly answered the Magnetic Field Line Question (with the lines not running through the points). One-quarter of the students answered the Magnetic Vector Question incorrectly and then answered the Magnetic Field Line Question correctly.

Of the students who took Version Two of the MGI pre-test, 35% of them answered both questions on the pre-test correctly. Fifty-five percent of the students answered both of the questions incorrectly. If we consider who put the same incorrect answer for both questions, 30% of the students responded that way. Practically no students answered the Magnetic Field Line Question (with the lines through the points) incorrectly and then went on to answer the Magnetic Vector Question correctly. Only 10% of the students correctly answered the Magnetic Field Line Question and then incorrectly answered the Magnetic Vector Question.

Commentary on Results:

As shown in Table 13, it appears that students struggle more with the Magnetic Vector Representation than with either type of Magnetic Field Line Representation. Furthermore, Table 14 indicates that slightly fewer students make the mistake of saying the field is zero inside the magnet on either version of the Magnetic Field Line Question than on the Magnetic Vector Question.

There is some evidence that field line representations help students correctly identify the direction of the magnetic field. On Version One of the pre-test, 25% of students incorrectly answered the Magnetic Vector Question but then were able to correctly answer the following Magnetic Field Line Question. This indicates that the Magnetic Field Line representation (even when the field lines do not run through the marked points) may help students understand the direction of the magnetic field. On Version Two of the pre-test, ten percent of students correctly answered the Magnetic Field Line Question and then incorrectly answered the Magnetic Vector Question. It appears that field lines are not confusing too many students. In fact, of the students who correctly answered the Magnetic Field Line Question on Version Two of the pre-test, 79% were able to correctly answer the Magnetic Vector Question. Field Lines do not seem to hurt student understanding; they may even improve understanding.

Overall, it seems that students struggle with identifying the correct direction of the magnetic field at various locations around a bar magnet, regardless of representation used. Only 15-35% of students answered both questions on the pre-test correctly (see Table 15).

To answer our question of consistency, however, we look at the percent of students who either answered both questions correctly or who put the same incorrect answer for both questions. On Version One of the pre-test, where the Magnetic Vector Question came first, 30% of students were consistent in their answers. On Version Two of the pre-test, where the Magnetic Field Line Question was first, a much larger percent of students, 65%, were consistent in their answers. It is tempting to interpret these results as telling us that students are more consistent when they see a field line representation before a vector representation. However, the effect we are seeing could also be due to the difference in Magnetic Field Line Questions between the two versions of the pre-test. (On one version the field lines ran through the points we asked about and on the other version the lines did not run through the points). To determine exactly what accounts for the difference in consistency, we will need to run two versions of the pre-test in the future in which we switch the Magnetic Field Line Questions used in the two versions from this past fall. In other words, Version One would have the Magnetic Vector Question followed by the Magnetic Field Line Question with the lines running through the points; Version Two would ask the Magnetic Field Line Question with the lines not running through the points and then ask the Magnetic Vector Question. Regardless of what is causing this difference in consistency, it appears that a large percent of students are not able to relate their answer on one representation type to their answer on the other representation type.

IV. DISCUSSION

In this thesis we explored student ability to relate a vector representation to an alternate representation of the same physical situation. The contexts included acceleration in one-dimension, electric fields, and magnetic fields. More specifically, in the context of acceleration we probed student ability to relate a vector representation of acceleration to a graphical representation (with both velocity vs. time graphs and acceleration vs. time graphs). For both the contexts of electric and magnetic fields we studied consistency between student responses to vector representations of the respective fields and their responses to field line representations.

The results of our research do not allow us to “rate” the representations we studied from “hardest” to “easiest.” For example, we cannot say that a vector representation of acceleration is harder for students to interpret than a vector representation of the electric field. We can’t make these claims because the contexts are very different and there are a unique set of challenges students face in each context. Our research question focused on student ability to relate representations *in* three different contexts. We have discussed the results for each context and some of the implications of those results in Chapter 3. We can make few universal claims based on our results.

One result that did persist regardless of the representation or context we were studying was that a common mistake students made was choosing a zero vector where the vector students were being asked about was not zero. The exact percent of students making this incorrect choice varied based on the representation they were being asked to consider, but it was always one of the most common errors. For example, on the acceleration in one-dimension vector question, a number of students said the acceleration was zero at the top of the ramp. Likewise, in the context of magnetic fields, many students incorrectly said that the field was zero inside the bar magnet. This was a common incorrect response for both the vector representation questions and the field line questions. This error seems to transcend all of the contexts we studied, and will need to be an area of focus for curriculum development in the future.

The other emergent theme we see is that students struggle to relate the representations we asked about in the three contexts we studied. In each of the contexts examined, students were asked about a vector representation and another representation type. We counted students' responses as consistent if they correctly answered both of these questions, or if they put the same incorrect response for both of these questions. Although the percent of students responding consistently varied based on context and the order in which questions were presented, overall there is much room for improvement in student consistency.

Of course, in order to correctly answer both questions, students must understand the representations individually. Our results show that within each context studied, some representations are more difficult for students to use than others. For example, it appears that students are better at interpreting magnetic field line representations than magnetic vector representations. In the future, curricular materials developed to improve student ability to relate representations will need to take the difficulty of individual representations into account. Each representation type may hold its own set of context-specific challenges that will need to be addressed before students can relate the representation to other representations. On the other hand, an interesting question for future research is if aiding students in understanding that their interpretations of two different representations of the same situation should be consistent, improves their understanding of the representations on their own.

The lack of consistent responses we saw for the representation types we tested is a problem because physics instructors require students to fluently use multiple representations and to comfortably go back and forth between different representation types. Moreover, representations are our way of capturing a real-world motion, process, or system on paper (or on the computer). If students cannot relate representations of a situation, they have some lack of understanding of the situation.

Materials should be developed that guide students through thinking about a vector representation of a situation and another way of representing that situation. This material should then force them to confront the idea that their interpretation of one representation type should be consistent with their interpretation of another representation type. The hope is that this material will increase student consistency, but recall students can be consistently *incorrect* (choose the same incorrect answer for both questions). Increasing the percent of students who are consistently *correct* will probably require additionally addressing other context-specific conceptual difficulties students have with the material being tested.

Within the “Commentary” sections of Chapter Three, some context-specific suggestions for future work were presented. I end this chapter with two broader ideas for future studies. First, in this thesis we only studied a handful of representation types. To obtain a more complete idea of student ability to relate vector representations to other representations, more representation types must be studied. Some of the more obvious next choices are strobe diagrams and written descriptions (which may come in the form of equations) of a situation. Second, there is still more to examine for the representation types discussed in this thesis. In particular, a detailed analysis of student reasoning for their answer choices would potentially illuminate why students fail to answer our questions consistently, and may point to instructional changes that would address these issues.

APPENDIX 1

Ramp Question run Fall 2014

This is only a preview of the survey. Responses will not be saved. Close

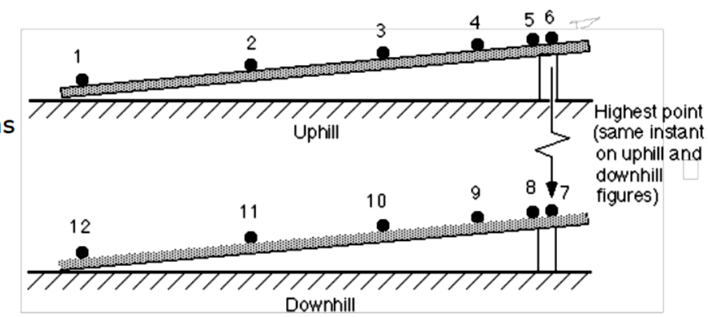
(A1D)U4d2A_Superposition
University of Washington

Page **3** of 4

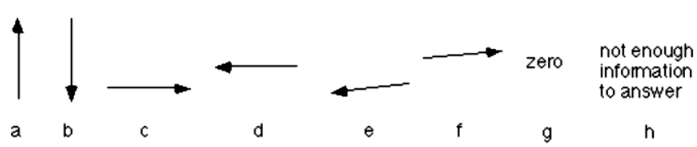
Animation End

WATCH THE ANIMATION AT RIGHT FOR PART A OF THE PRETEST (the animation will run once).
A ball is given a quick push such that it rolls up and then down a ramp. The motion of the ball **when the hand is no longer pushing the ball** is demonstrated at right.

Part A: The strobe diagram at right represents the motion of the ball as it rolls up and then down the track. (In a strobe diagram, the position of an object is shown at instants separated by equal time intervals.)



For the following questions, choose the arrow from the list below that best represents the direction of the acceleration of the ball.

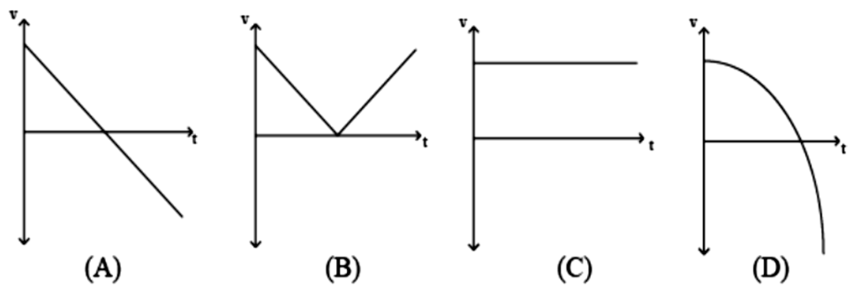


Question 8.
For each of the following locations of the ball, which of the arrows above best represents the instantaneous acceleration of the ball?

	a	b	c	d	e	f	g	h
Location 3	○	○	○	○	○	○	○	○
Location 6	○	○	○	○	○	○	○	○
Location 10	○	○	○	○	○	○	○	○

Question 9.
Explain the reasoning you used to obtain the direction of the instantaneous acceleration of the ball at each location.

Question 10.
Which of the following velocity vs. time graphs best represents the motion of the ball as it moves up and then down the ramp? Assume that the positive direction corresponds to motion up the ramp.



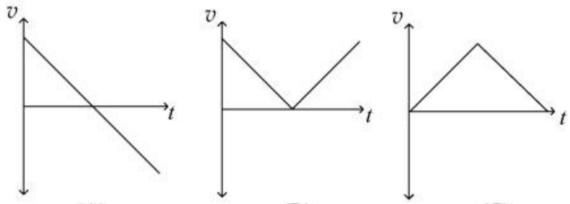
(E) None of the above

- ○ A.
- ○ B.
- ○ C.
- ○ D.
- ○ E. None of the above

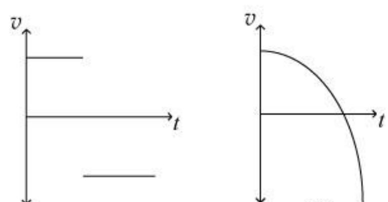
Question 11.
Briefly explain your reasoning for your previous response.

Modifications of Ramp Question run Winter 2015
(Only the Graph question changed).
Version A:

Question11. Which of the following ***velocity*** vs. ***time*** graphs best represents the motion of the ball as it moves up and then down the ramp? Assume that the positive direction corresponds to motion up the ramp.



(A) (B) (C)

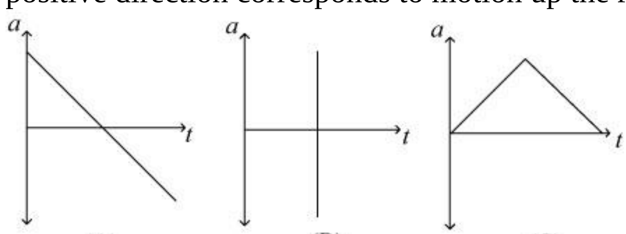


(D) (E)

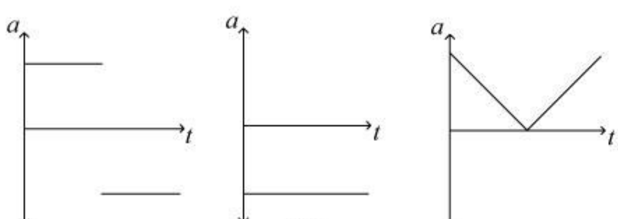
(F) None of the above

Version B:

Question 11. Which of the following ***acceleration*** *vs.* ***time*** graphs best represents the motion of the ball as it moves up and then down the ramp? Assume that the positive direction corresponds to motion up the ramp.



(A) (B) (C)



(D) (E) (F)

(G) None of the above

APPENDIX 2: EPD

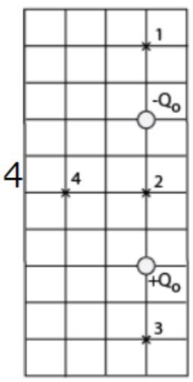
Fall 2014

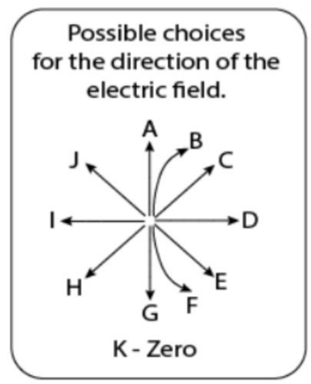
Version 1:

(EPD)U3b3

University of Washington

A. Two point charges, $+Q_0$ and $-Q_0$, are fixed in place at the positions shown. Four locations 1, 2, 3, and 4 are each marked with an 'x.'





Question 1.

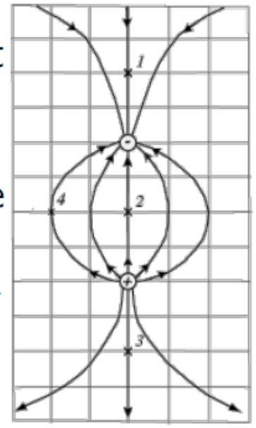
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? Select option K (zero) if the field is zero.

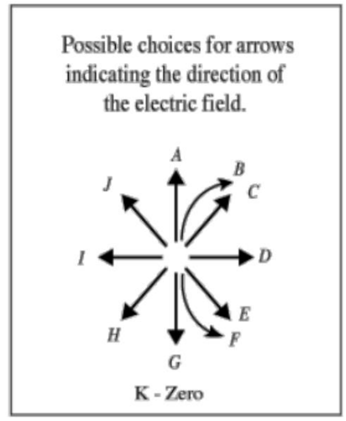
	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 2.

Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

B. The diagram at right shows electric field lines that correspond to two charges that are fixed in place. Four locations 1, 2, 3, and 4 are each marked with an 'x.'





Question 3.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? Select option K (zero) if the field is zero.

	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

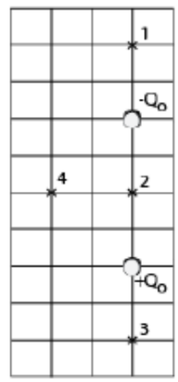
Question 4.
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

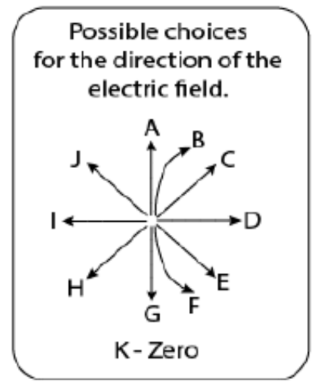
Version 2:

(EPD)U3b4
University of Washington

Page **1** of 3

A. Two point charges, $+Q_o$ and $-Q_o$, are fixed in place at the positions shown. Four locations 1, 2, 3, and 4 are each marked with an 'x.'



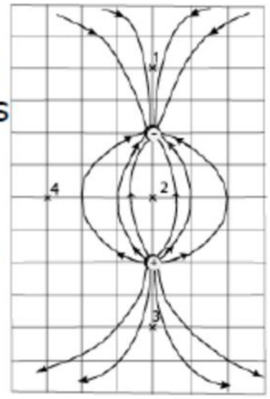


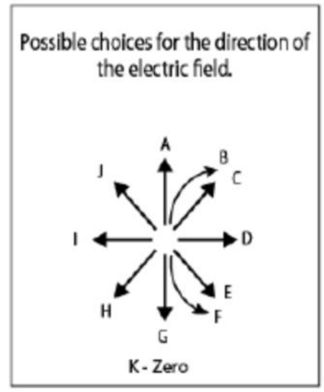
Question 1.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? Select option K (zero) if the field is zero.

	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 2.
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

B. The diagram at right shows electric field lines that correspond to two charges that are fixed in place. Four locations 1, 2, 3, and 4 are each marked with an 'x.'





Question 3.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? Select option K (zero) if the field is zero.

	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1												
Location 2												
Location 3												
Location 4												

Question 4.
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

EPD Winter 2015
Version 3

(EPD)U3c_R1
University of Washington

Time remaining:
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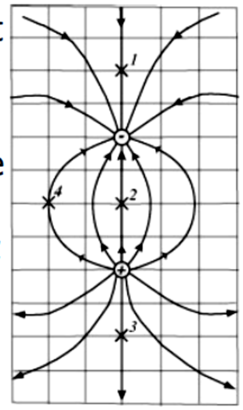
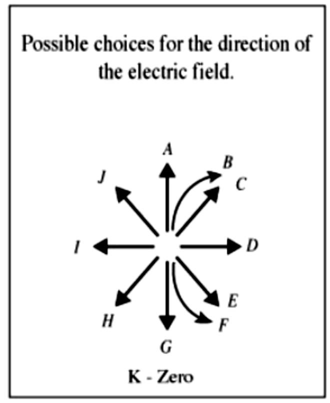
Page **1** of 4

A. The diagram at right shows electric field lines that correspond to two charges that are fixed in place. Four locations *1, 2, 3,* and *4* are each marked with an 'x.'



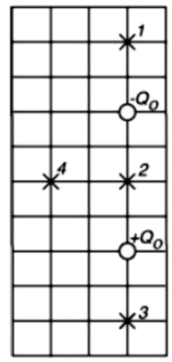


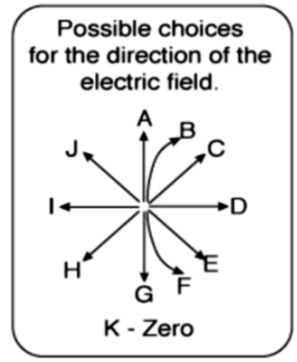
Question 1.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? If the field is zero, select option K.

	A	B	C	D	E	F	G	H	I	J	K (Zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 2.
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

B. Two point charges, $+Q_o$ and $-Q_o$, are fixed in place at the positions shown. Four locations 1, 2, 3, and 4 are each marked with an 'x.'





Question 3.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? If the field is zero, select option K.

	A	B	C	D	E	F	G	H	I	J	K (Zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 4.

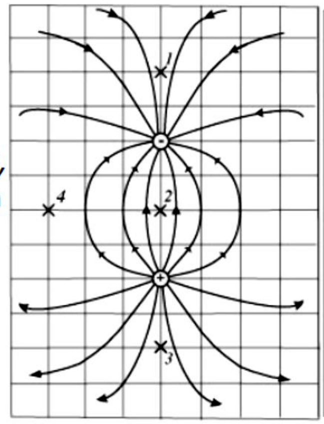
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

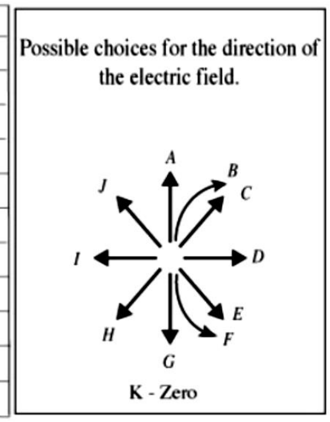
Version 4

(EPD)U3c_R3

University of Washington

A. The diagram at right shows electric field lines that correspond to two charges that are fixed in place. Four locations *1, 2, 3,* and *4* are each marked with an 'x.'



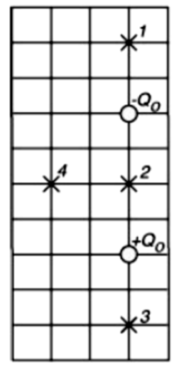


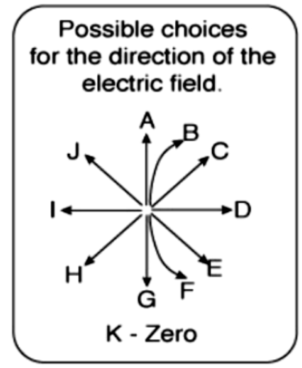
Question 1.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? If the field is zero, select option K.

	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 2.
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

B. Two point charges, $+Q_0$ and $-Q_0$, are fixed in place at the positions shown. Four locations 1, 2, 3, and 4 are each marked with an 'x.'





Question 3.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? If the field is zero, select option K.

	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 4.

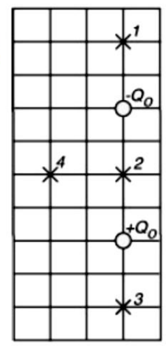
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

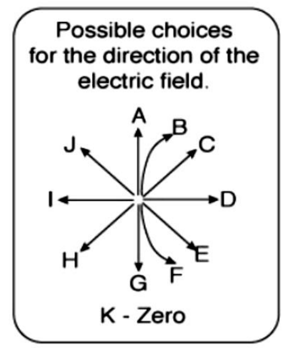
Version 5

(EPD)U3c_R4

University of Washington

A. Two point charges, $+Q_0$ and $-Q_0$, are fixed in place at the positions shown. Four locations *1, 2, 3,* and *4* are each marked with an 'x.'





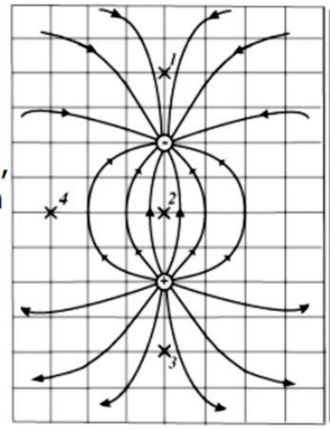
Question 1.
Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? If the field is zero, select option K.

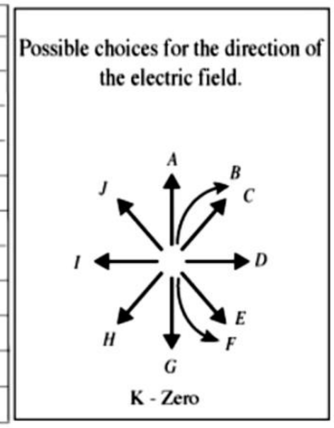
	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1	○	○	○	○	○	○	○	○	○	○	○	○
Location 2	○	○	○	○	○	○	○	○	○	○	○	○
Location 3	○	○	○	○	○	○	○	○	○	○	○	○
Location 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 2.

Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

B. The diagram at right shows electric field lines that correspond to two charges that are fixed in place. Four locations 1, 2, 3, and 4 are each marked with an 'x.'





Question 3.

Which choice (A-K, shown above right) best represents the direction of the electric field vector at the following locations? If the field is zero, select option K.

	A	B	C	D	E	F	G	H	I	J	K (zero)	None of the choices above accurately show the direction of the electric field vector at this location.
Location 1												
Location 2												
Location 3												
Location 4												

Question 4.

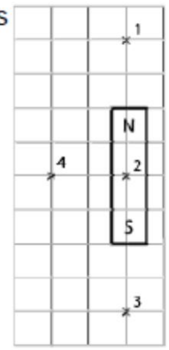
Explain your reasoning for your answers for each of the four locations 1, 2, 3, and 4.

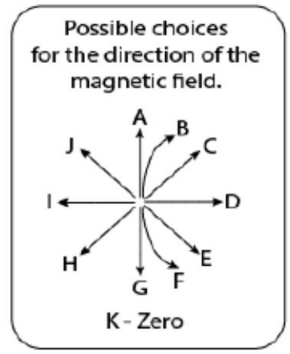
APPENDIX 3

MGI Pre-test, Version 1

(MGI)U2e1
University of Washington

A bar magnet is shown at right. Four locations near the magnet *(1, 2, 3,* and *4)* are each marked with an 'x'. Points *1, 2,* and *3* all lie along the axis through the center of the magnet. Point 2 lies *inside* the magnet.





Question 1.

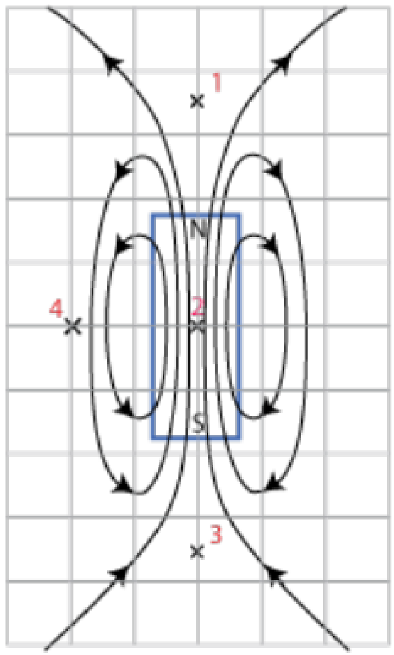
Select the arrow from the figure (above right) that most accurately indicates the direction of the magnetic field vector due to the bar magnet at each point below. If the field is zero at a given point, select option K (zero) for that point.

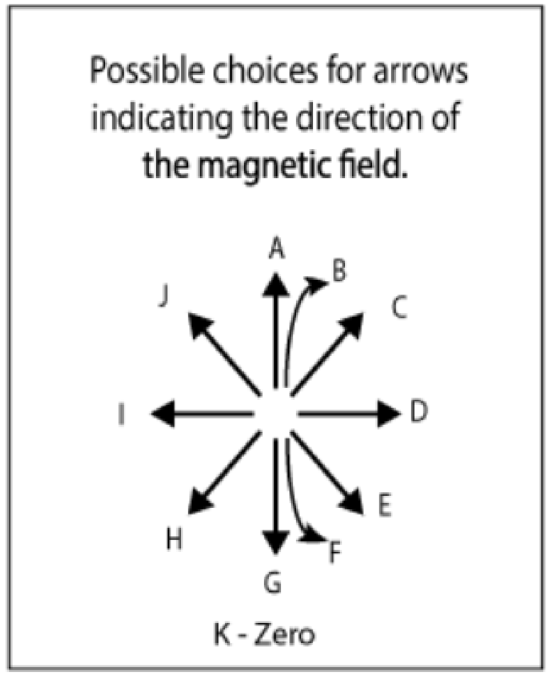
	A	B	C	D	E	F	G	H	I	J	K (zero)	L (none of the above)
Point 1	○	○	○	○	○	○	○	○	○	○	○	○
Point 2	○	○	○	○	○	○	○	○	○	○	○	○
Point 3	○	○	○	○	○	○	○	○	○	○	○	○
Point 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 2.

Explain the reasoning you used to obtain your answer to the previous question.

The diagram below shows magnetic field lines that correspond to a bar magnet (outlined in blue). Four locations near the magnet *(1, 2, 3,* and *4)* are each marked with an 'x'. Points *1, 2,* and *3* all lie along the axis through the center of the magnet. Point 2 lies *inside* the magnet.





Question 3.
Select the arrow from the figure (above right) that most accurately indicates the direction of the magnetic field vector due to the bar magnet at each point below. If the field is zero at a given point, select option K (zero) for that point.

	A	B	C	D	E	F	G	H	I	J	K (zero)	L (none of the above)
Point 1	○	○	○	○	○	○	○	○	○	○	○	○
Point 2	○	○	○	○	○	○	○	○	○	○	○	○
Point 3	○	○	○	○	○	○	○	○	○	○	○	○
Point 4	○	○	○	○	○	○	○	○	○	○	○	○

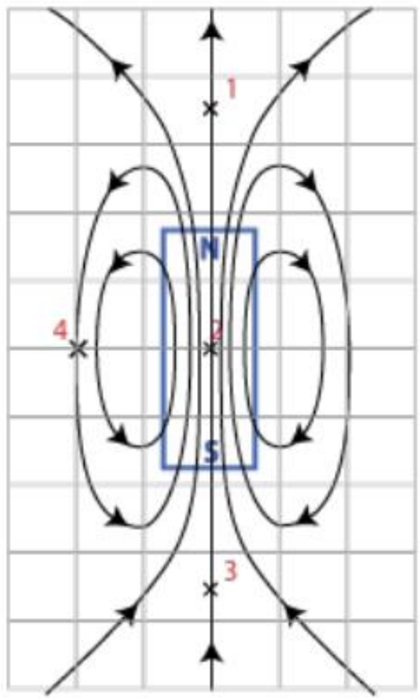
Question 4.

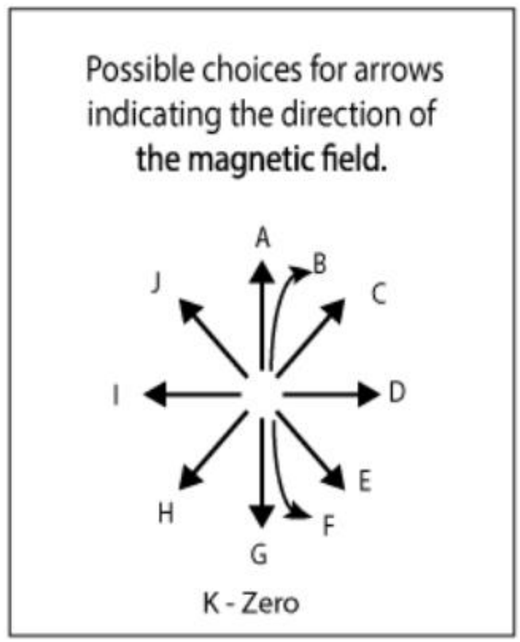
Explain the reasoning you used to obtain your answer to the previous question.

MGI Pre-test, Version 2

(MGI)U2e2
University of Washington

The diagram below shows magnetic field lines that correspond to a bar magnet (outlined in blue). Four locations near the magnet *(1, 2, 3,* and *4)* are each marked with an 'x'. Points *1, 2,* and *3* all lie along the axis through the center of the magnet. Point *2* lies *inside* the magnet.



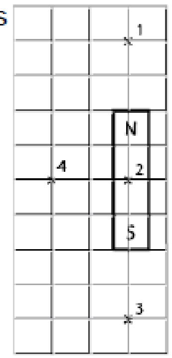


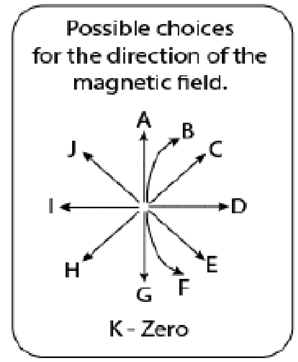
Question 1.
Select the arrow from the figure (above right) that most accurately indicates the direction of the magnetic field vector due to the bar magnet at each point below. If the field is zero at a given point, select option K (zero) for that point.

	A	B	C	D	E	F	G	H	I	J	K (zero)	L (none of the above)
Point 1	○	○	○	○	○	○	○	○	○	○	○	○
Point 2	○	○	○	○	○	○	○	○	○	○	○	○
Point 3	○	○	○	○	○	○	○	○	○	○	○	○
Point 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 2.
Explain the reasoning you used to obtain your answer to the previous question.

A bar magnet is shown at right. Four locations near the magnet *(1, 2, 3,* and *4)* are each marked with an 'x'. Points *1, 2,* and *3* all lie along the axis through the center of the magnet. Point 2 lies *inside* the magnet.





Question 3.

Select the arrow from the figure (above right) that most accurately indicates the direction of the magnetic field vector due to the bar magnet at each point below. If the field is zero at a given point, select option K (zero) for that point.

	A	B	C	D	E	F	G	H	I	J	K (zero)	L (none of the above)
Point 1	○	○	○	○	○	○	○	○	○	○	○	○
Point 2	○	○	○	○	○	○	○	○	○	○	○	○
Point 3	○	○	○	○	○	○	○	○	○	○	○	○
Point 4	○	○	○	○	○	○	○	○	○	○	○	○

Question 4.

Explain the reasoning you used to obtain your answer to the previous question.

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