Mount Holyoke College

On Intersecting 2-Brane Solutions in Type IIA Supergravity Theory

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Physics

I'd like to dedicate this research paper to Dr. Moataz Emam who has humored me for this whole year. Allowing me to explore the incredible and phenomenonal field of string theory and allowing me to write a theory thesis as an undergraduate.

ACKNOWLEDGEMENTS

- String theory is not a subject that is typically seen in an undergraduate program. I would like to thank my advisor, Moataz Emam, for taking a chance with me and allowing me to explore String Theory to the fullest of my capabilities. For allowing me to question constantly and to learn more in a year than I thought I was capable of.

- College is a unique experience to each individual and I would like to thank Mount Holyoke's Physics Department for being incredibly supportive in all my endeavors. For allowing me to have an independent study in physics theory research my first year, which initially sparked my curiosity and has now lead me to write a research paper on a similar [but not quite the same] topic four years later. Being a part of this department has been an amazing experience, it inspires me more each and every year.

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- And I'd like to thank my mother for pushing me when I didn't think I could do it. For always having faith and showing me that hard work and persistence does pay off!

- Lastly, I would like to thank all the women here who have kept me sane while I have been doing my research and listening to me babble on for hours about something they know absolutely nothing about.

ABSTRACT

On Intersecting 2-Brane Solutions in Type IIA Supergravity Theory

May 2005

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In string theory, a string is a one-dimensional strand of energy with a size scale of 10^{-33} cm living in ten dimensional spacetime. Theoretically, the known subatomic particles, such as quarks and electrons, are made of strings, where their properties, such as mass and spin, are consequences of different oscillations on the string. The appeal of string theory arises from the fact that it also contains within it a quantum mechanical version of general relativity as a special case, providing for the first time the possibility of a theory of all the known interactions.

Supergravity theories arise from string theory as low energy limits. They are classical theories of gravity and other types of fields. Solutions to the nonlinear Einstein and field equations of supergravity are necessarily solutions of string theory as well. Many such solutions are known as *p*-branes: extended objects spanning *p* spatial dimensions. A 0-brane is a point-like object while a 1-brane is string-like and so on. The hunt for and categorization of such solutions has been particularly popular in the literature in recent years.

We are interested in a solution in type IIA supergravity that corresponds to two intersecting 2-branes coupled to a 3-form gauge potential, and a scalar field traditionally known as the dilaton. We derive the equations of motion of the theory and construct an ansatz to the solution. Other, previously known, intersecting brane solutions[1] have provided the groundwork for further understanding the full structure of string theory and how it relates to other theories in lower dimensions.

We want to know this because we want to classify as many possible solutions to string theory as are possible. Classifying these solutions may lead to understanding some cosmological applications of branes. Brane solutions may also provide us clues to understanding nonlinear theories in general. These solutions are not limited to string theory they may even lead to applications in particle physics theory as well.

[1] Such as Nucl. Phys. **B** 478:544-560, 1996 and Phys. Rev. **D** 63:064003, 2001

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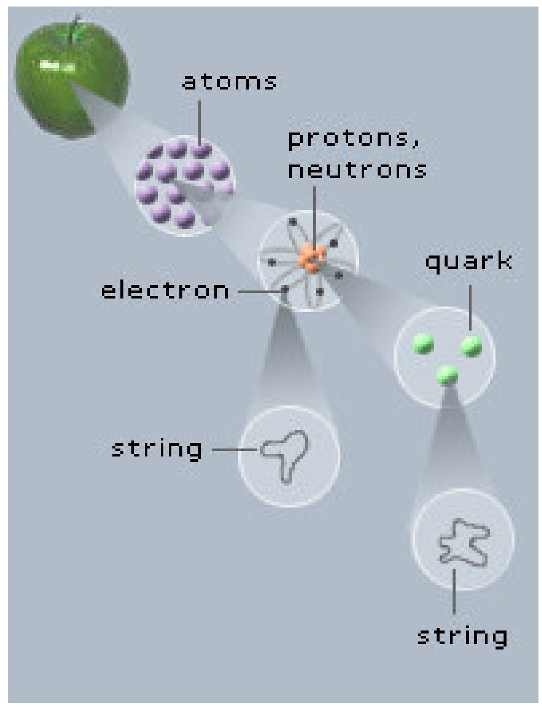
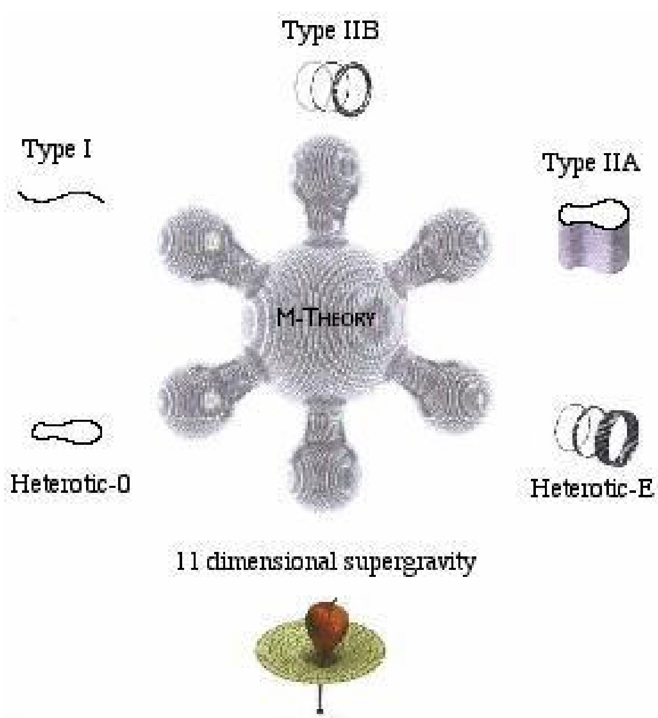
I. INTRODUCTION

Now you may be asking yourself, what is this String Theory you speak of? Are you talking about little pieces of string floating around in space? How does that work? And why do we care about this elusive String Theory? Well it's not exactly just some little pieces of string floating around in space. It is believed that strings are tiny one-dimensional strands of energy on a scale of approximately 10^{-33} centimeters that are like oscillating filaments and are contained within the smallest known particles of nature, such as quarks. The reason for having string theory is to provide the complete unification of physics and to bridge the gap between Quantum Mechanics and General Relativity because as it stands both may not work at the same time.



The major conflict in modern physics is that when you try to make general relativity quantum mechanical the theory blows up. Both cannot be right because when you put the two together you get an infinite number of infinite solutions (it is a non-renormalizable) and we know that that is impossible. So basically the two most important theories in modern physics, one for big things and the other for little ones, are mutually incompatible. How is it that these two very important theories have gone ununified for such a long time? It's actually quite simple, in general relativity you study massive objects in the cosmos and in quantum mechanics you are studying tiny particles that are not visible without some sort of aid. Therefore it is quite plausible that you don't *need* to use both general relativity and quantum mechanics at the same time. Since it is not an immediate concern that absolutely needs to be resolved, research has continued using both of these theories without any problems. For particles (atoms and the like) we have quantum mechanics. For stars and galaxies we have general relativity. It's not perfect but it works, right? That would be the thought but in fact we did find some cases where it is a problem. There's always an extreme case, for example a black hole or the big bang. A black hole is a massive object that is crushed down to a miniscule size while the Big Bang is a miniscule object that rapidly expanded into this universe – this would require the use of both general relativity and quantum mechanics. But wait, you can't do that! When you try to use the two theories together they give you results that mean nothing. Now, many physicists and mathematicians have been hard at work looking at what exactly makes up matter so that we would no longer have such problems. And they have come up with a fabulous new theory called superstring theory[2]. The best part about this new theory is that not only does it resolve the tension between the two older more well known theories but it actually requires



them to use one another for it to work! String theory appears to have the potential to actually be the unified field theory, the theory of everything, the master equation. [As an aside, string theory is actually seen as a special case of a larger more general theory named M-Theory. It is thought that the string theories are simply special cases of M-Theory

[2] Such physicists include: Michael Green, John Schwarz, David Gross are a few of the first string theorists.

such that each solution we find in string theory brings us closer to M-theory.]

Now the reason why this is so interesting for modern physics is that String Theory is trying to be the "unified field theory" that Einstein had envisioned. The reason that String Theory is able to do this is because the core of this theory is that everything is just different combinations of vibrating strands, or strings if you will. If everything is just different combinations of vibrating strands then it must follow that the theory that explains these strings will be the theory for everything else. Think of these vibrating filaments just as you would normal strings that oscillate with resonant frequencies, only they do not produce notes but rather the particles' charges and spins.[3] So far the theory that has been constructed appears to require ten spacetime dimensions. An important thing to note is that there are five different types of string theory: Type II A string theory, Type II B string theory, Type I string theory, SO(32) heterotic string theory, and E_8xE_8 heterotic string theory. How could the one so-called theory of everything come in five different theories? The five are quite similar, though they differ in the minor intricacies of the theories. Type I string theory includes both open and closed strings. This theory resembles type IIB theory except for the lack of open loops in the type IIB theory. In this theory, the clockwise and counterclockwise vibrations of the strings are opposite. The difference between clockwise and counterclockwise vibrations is from the reference point we choose to look at, whether we choose to take the bosonic oscillations or the fermionic oscillations as our reference point. Whichever one we choose to be our reference point will indicate which direction we find our vibrations. Another way of describing this idea is stating that particles involved with this theory spin in different directions. Type IIB is the opposite in nature of the type IIA theory, the type IIB string theory has clockwise and counterclockwise vibrations that are the same. Similarly, the spin of the particles is identical in this theory. Heterotic string theory is very interesting because it combines bosonic strings, which need twenty-six spacetime dimensions, and supersymmetry, which involves ten dimensions. This combination results due to the vibrational patterns. Counterclockwise vibrational patterns occupy twenty-six spacetime dimensions, while clockwise vibrational patterns occupy ten spacetime dimensions. This means that the additional sixteen space dimensions are somehow condensed into a circular shape. Because there are two shapes that this circular structure can take, heterotic type SO(32) [HO] and heterotic type E_8 X E_8 [HE] theory emerged to account for these two possibilities.[4] The type E_8 X E_8 [HE] theory is the one that possibly contains the standard model as a special case.

[3] http://www.pbs.org/wgbh/nova/elegant/everything.html
[4] http://library.thinkquest.org/04apr/01330/newphysics/typesof_st.htm

II. Background

a. Back to the Basics

Before we start with string theory, we need to understand where the importance of it lies and before we can do that we need to look at some physics history first. So, the three major conflicts that have occurred in physics are Newton vs. Maxwell, Newton vs. Gravity and Einstein vs. Quantum. So the first one would be Newton's laws of motion that allow one to theoretically be able to run up to the speed of light, where, according to Maxwell's theories of Electromagnetism, this is not possible. This lead to Einstein's theory of special relativity that took the difficulties involving the speed of light and made light relativistic in that it does not change its speed ever and nothing may travel at its speed. Maybe for different vantage points there are differing experiences, meaning that there is no set space and time but that they are malleable constructs. Unfortunately, this lead to more conflict, the one between Newton's experiments and theories about gravitation and how it "involves influences that are transmitted over vast distances of space instantaneously". Again, Einstein solved this problem with his theory of general relativity. But there was yet another difficulty to face and this was when physicists began developing quantum mechanics they realized that Einstein's general relativity is not compatible with their new quantum mechanics. Only this time there was no solution. That is until string theory came along!

Originally the Greeks thought that the smallest possible particle was the atom [something uncuttable] but they were unfortunately mistaken as was later proved with the discovery of the proton, electron and quarks and then with even more tiny particles to follow. There is no evidence that electrons or quarks are made up of smaller particles but what we do see is that the universe as a whole has many other "particulate ingredients" such as a photon. From this point on I'm assuming some physical background and not going into what the other particles are that may be mentioned later on in this paper. Why do these other particles, such as photons, exist? They exist because our world is made up of forces, which are made up of smaller particles, such as photons. A photon is a tiny bundle of the electromagnetic force. So these tiny bundles [or better known as particles] are the smallest possible bundles of these extremely important forces that govern our universe. The four main forces are the electromagnetic force, the strong force, the weak force and gravity. Each of these have tiny bundles that match them up, as I mentioned earlier electromagnetism has the photon, the strong force has gluons [it's what makes the nucleus of an atom stay together], the weak force has weak gauge bosons and gravity theoretically has the graviton. The reason I say theoretically for the graviton is because it hasn't quite been observed yet and for good reason, gravity is a very feeble force so trying to find the smallest packet of the feeblest force is really quite difficult to do. These forces are very important and they are all interdependent. For example the strong force and the electromagnetic force rely on each other in order for the nuclei of atoms to stay together. So a relatively small change in either of these forces could throw our entire universe out of whack. For example, if the electromagnetic repulsion threw over the strong force binding the atomic nuclei together it's possible that none of the elements we know would exist. If our elements don't exist then we don't exist. Just one of many ways that we know these forces are incredibly important to our own lives.

b. HOW DOES STRING THEORY FIT IN?

So how does string theory fit into all of this? String theory tries to explain why matter and forces are the way they are in the universe. It sets up a framework to answer all of the questions we have about how these forces interact and work together. So first let's figure out what we mean by a string. Well we're looking at these tiny particles right, but that's not all there has to be something more to them, right? Well we find that when you look very closely at them that they are not point-like particles but instead that they are tiny one dimensional loops or open strings of energy on the scale of 10^{-33} cm. "Like an infinitely thin rubber band, each particle contains a vibrating, oscillating, dancing filament that physicists, […], have named a *string*."[5] So since these are oscillating strings they have the same properties as the strings that we know and love. You may ask, why do we care that they have the same oscillating principles? Well, my friend, if a string oscillates like a violin string then it must have a resonance frequency just as a violin string. Unlike a musical string the resonance will not produce a musical note, this string's resonance pattern corresponds to its particle's mass or spin! Basically all this universe boils down to is a bunch of infinitesimal strings that oscillate in different patterns to make up what we are and the world that we are living in. Yes, this is a bit of a stark view of life, but if you want to understand it, it is quite beautiful. So the big worry on this one would be: if this is the Theory of Everything, where do we go from here? Is all the research over? No not at all, this is simply a means to begin figuring out just how these strings affect our world and it gives us a better understanding with which to further our research endeavors. Do take note that string theory is still a work in progress there is much to learn and be learned about it.

c. Why was Newton a Problem?

How did string theory come about? Well to understand that we have to look back on the three major conflicts of some very well established physical theories that I mentioned earlier.

[5] Greene,

The first major conflict revolves completely around light. According to Newton's laws of motion one should be able to catch up to the speed of light so that when they do they will appear to stand still but according to Maxwell [and our own observations] that isn't really possible. There is no stationary light, you can't hold light in the palm of your hands, and so what do we do with this pesky little problem with light? Thankfully Einstein was also very troubled by this problem and ended up creating a theory that we now know as special relativity to resolve this matter. He changed our entire view of how we look at space and time. They are no longer fixed but rather are relative to each individual. With special relativity we know that there are very often miniscule differences in how one observer views an event from another because of his/her viewpoint. The reason that it is so miniscule is because the speed of light is so much greater than anything we normally would experience that the differences that are there are generally negligible. The main purpose of special relativity is to understand that "motion is relative". "No matter how hard you chase after a light beam, it still retreats from you at light speed." How is that possible? "Things that are simultaneous from the viewpoint of some observers will not be simultaneous from the viewpoint of others, if the two groups are in relative motion." An example that Greene uses in the Elegant Universe is that of observers watching two men sign a paper on a train from the platform (note that the men have to be spatially separated). From the perspective of those on the train, the men signed the piece of paper simultaneously whereas from the perspective of the platform there was one who signed first and one who signed second. The importance of this would be that the photons traveling from the train appear different to those in a different relativistic perspective. He also uses the example of a light clock to show us the distortion of time because there is more to special relativity than simply distortion in space but also distortion in time. So for a light clock that is in motion it appears that the time is off by a miniscule amount, but this amount is so tiny that we do not notice it. These are simply some examples of the distortion of motion and time due to relativity. The reason that this is not more common knowledge is because the distortion both due to space and time is so small that one does not observe this with the instruments we use to measure these observations. So along with these other observations we take a look at a muon, or two to be exact: a muon in motion and a muon at rest. If there were to be a muon in motion it may have a longer "lifetime" than the muon at rest because, as we keep seeing with special relativity, motion changes time. Granted the moving muon may have a longer life-span but it will not accomplish any more than the muon at rest because even though its life is technically longer, all of its other capacities have been slowed down by time as well, so it cannot do more with the length of time that has changed. To continue with Greene's example of a muon, what we find is that since time is warped that if a muon were to read a book during its lifetime its processes would be slowed down so that the muon at rest would read the same number of pages in the book as the muon at rest because of the space-time warp. Basically, special relativity accounts for the change of space and time so one can be moving and things will be different from what is "experienced". **"Time is affected by motion"!** So from all of this talk of special relativity we finally concede that time is actually another dimension, the fourth dimension. And we must remember that there is no passage of time at the speed of light. So, you say, why can't we make something move at the speed of light? Well, the answer is relatively simple; it's due to Einstein's equation *[* $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$, so that as $v \to \infty,\ \ m \to \infty$ *]*, the more massive an object is the harder it is to speed up. And a muon traveling at 99.9% the speed of light weighs almost 22 times more than muons at rest. Eventually what happens is that the muon in motion will become infinitely heavy and therefore it will take an infinite amount of energy to speed up, which is impossible. Therefore this shows that nothing can travel faster than the speed of light.

We have now found that nothing can outrun light, but what about gravity? According to Newton there can be instantaneous gravitational changes, how can that be possible if nothing can travel faster than the speed of light? Einstein solved this Newtonian glitch, being able to catch up to the speed of light with Special Relativity, but he hadn't taken Gravity into account yet, so how do we account for that, since Special Relativity didn't fix that problem. Once again we have Einstein to thank for our solution to this problem with his theory of General Relativity, which tells us that with accelerated reference frames we can make up for gravity because they are connected to one another. What this means is that a person in a gravitational field cannot distinguish between gravity and acceleration. You may relate this to traveling in an elevator, we feel a force pushing us downward as the elevator travels upward but we cannot distinguish whether it is the effect of gravity or the acceleration of the elevator traveling upward while we are in the elevator. Essentially we are looking at the universe from an accelerated reference point, which is different from the vantage points used in Special Relativity. What we are proposing is that there is a spacetime fabric that can be curved and bent, generally warped due to this state of accelerated motion. "Time is warped if its rate of passage differs from one location to another." This stems from Newton's statement that "Gravity must be caused by an agent", whose agent is the spacetime "fabric of the cosmos" according to Einstein. Therefore, the warping of space is gravity and if the space is warped then the path is curved, which makes sense from what we have previously learned. We don't feel space when we are immersed within its fabric because we feel gravity which uses space as its medium. Since space and time are now not static concepts we have resolved special relativity with the gravitational force and turned it into general relativity. Essentially "gravitational disturbances keep pace with, but do not outrun, photons". The fabric of the universe is stretching or it is shrinking, but it is not staying put. The equations of general relativity show this explicitly in the Einstein equation $G_{MN} = 8\pi T_{MN}$ (tensor indices run over 0,…,3; 3 space dimensions & 1 time dimension) where the left hand side is geometry and the right hand side is matter content.

d. THE REAL DEAL ON UNCERTAINTY

Now that we have an understanding of general relativity we need to move on to Quantum Mechanics. On an extremely small scale things such as walking through walls and objects disappearing (better known as quantum tunneling) are quite common whereas on a large scale, what we can readily see, this is impossible. The first challenge to figuring out quantum theory was in the original assumption that, since there are an infinite number of wave patterns that carry the same amount of energy, this leads to an infinite amount of energy for a defined space. This is a problem because you can't really have an infinite amount of energy in any one space, or rather with this logic in all spaces because we can clearly see that there are certain things that contain more energy than others. This was resolved when Planck figured out that there is a minimum energy a wave can carry, which is proportional to its frequency, but at some point this minimum will be too large for the space we have defined and therefore will limit the number of wave patterns in our space. But we're still saying that they travel in waves, right? And everything seems to be smooth? So what's all this talk of minimum energy and stuff like that? What Planck really discovered is that this energy comes in discrete packets but these steps are so small that when we observe them it looks like a smooth wave.

This gets explained more when we look at the photoelectric effect, where you would think that when the intensity of the light emitted increases that the speed of the electrons emitted would increase too. But this is not the case, what really happens is that the number of electrons ejected increases but their speed doesn't change. On the other hand, if the frequency of the light changes the speed of the emitted electrons will change as well. Essentially, it is the frequency of the light not the total energy that determines whether electrons are ejected and what energy they have. Now if we take this with Planck's theory of energy being distributed in packets then we find photons, or tiny packets of light which creates the wave-particle phenomena because they act as particles but together we see them as waves. We can see this experimentally through the double-slit experiment and the photoelectric effect because the double-slit experiment shows light to have wave-like properties whereas the photoelectric effect shows them to have particle-like properties. This is still troubling because how can one phenomenon have two contradicting properties. This becomes more apparent with the discussion of probability. Electron waves give us an idea of where they are *likely* to be not where they *are*. This becomes apparent with Heisenberg's uncertainty principle, where it is not possible to measure both a particle's position and its velocity at the same time, or similarly you cannot know a particle's energy and momentum at the same time.

e. OH STRING THEORY, HOW DO I LOVE THEE, LET ME COUNT THE WAYS…

Simply put the major conflict between General Relativity and Quantum Mechanics is that General Relativity is based on the belief of a smooth spatial geometry whereas Quantum Mechanics has huge fluctuations on a small spatial scale. In this sense quantum mechanics' central feature being the uncertainty principle is in direct conflict with general relativity's central principle which is the smooth geometrical model of space and of spacetime. When we try to show this mathematically and merge the equations together you get infinite solutions, which is just wrong. But this becomes unified through string theory because string theory is such a flexible theory! Each string has a resonance frequency and these resonances "give rise to different masses and force charges". This unifies the two theories because through the resonant patterns of the different forces so we can understand general relativity and quantum mechanics in one unique theory of everything!

Let's do a little history on string theory. String Theory really came into existence in the 1970's when particle theorists discovered that there were dual theories that describe the particle spectrum that also describe the quantum mechanics of oscillating strings. Coincidence? I think not. Just after that physicists discovered Supersymmetry. Supersymmetry is important because we need to have Supersymmetry observed for String Theory to hold true. What Supersymmetry claims is that there is symmetry to everything, in that if we have a particle with an integer spin (bosons) then there must be a partner to it somewhere that has a half-integer spin (fermions). I am not going to go into any more detail with this because it is unnecessary for my solution, we will simply assume that Supersymmetry is maintained within the problem due to similar problems that have been done in the past. There will be some indications in my calculation that supersymmetry is preserved.

One solution of ordinary general relativity that may be helpful before we get into more detail would be the Schwarzschild solution that describes a black hole.

$$(\text{II.1}) \qquad ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2$$

$$(\text{II.2}) \qquad B(r) = \left(1 - \frac{2MG}{c^2 r}\right)$$

$$(\text{II.3}) \qquad A(r) = \left(1 - \frac{2MG}{c^2 r}\right)^{-1}$$

$$(\text{II.4}) \qquad ds^2 = \left(1 - \frac{2MG}{c^2 r}\right) dt^2 - \left(1 - \frac{2MG}{c^2 r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2$$

r, θ, ϕ : spherical coordinates in 3 dimensions

G is Newton's gravitational constant

M is the mass & c is the speed of light

This metric (a generalized Pythagorean theorem in spacetime) describes how space and time warp when we look at a black hole in that if something gets too close to a black hole it will be unable to escape its gravitational grip and it will be inevitably destroyed because of the magnitude of the gravitational force on it. Also when one is in that proximity to a black hole time slows down because of the laws of General Relativity. This problem with the black hole is similar to that of the Big Bang in that it is the exact opposite of what happened. In theory, the Big Bang was a point that had so much energy that it exploded and created the universe that we now live in, whereas a black hole takes any mass from this universe and will compact it down to almost nothing.

Brane solutions of supergravity are simply generalized solutions of charged black holes whose action appears as:

$$(\text{II.5}) \qquad S = \int d^4x\sqrt{-g}\,(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu})$$

g is the determinant of the metric

$F_{\mu\nu}$ is the electromagnetic field tensor

R is the Ricci scalar that we get from:

$$(\text{II.6}) \qquad R = g^{\mu\nu}R_{\mu\nu} = \left(\partial_\alpha \Gamma^\alpha_{\mu\nu}\right) - \left(\partial_\nu \Gamma^\alpha_{\mu\alpha}\right) + \Gamma^\beta_{\mu\nu}\Gamma^\alpha_{\beta\alpha} - \Gamma^\beta_{\mu\alpha}\Gamma^\alpha_{\beta\nu}$$

where the Christoffel symbols are as follows,

$$(\text{II.7}) \qquad \Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left[\left(\partial_\mu g_{\beta\nu}\right) + \left(\partial_\nu g_{\beta\mu}\right) - \left(\partial_\beta g_{\mu\nu}\right)\right]$$

The simplest solution to the black hole would be the Reissner-Nordstrøm solution, which is a static, electrically charged black hole, that has a mass M and an electric charge Q.

$$(\text{II.8}) \qquad ds^2 = -fdt^2 + f^{-1}dr^2 + r^2 d\Omega^2$$

$$(\text{II.9}) \qquad A_t = \frac{Q}{r}$$

where,

$$(\text{II.10}) \qquad f = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)$$

and $d\Omega^2$ is the metric of a unit sphere

$$(\text{II.11}) \qquad d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$$

which gives us:

$$(\text{II.12}) \qquad ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

As you can see this will reduce down to the Schwarzschild metric if we set $Q = 0$. A_t is the Coulomb potential. Also note here that if $Q = M$, the Reissner-Nordstrøm solution can be written in the simple form:

$$(\text{II.13}) \qquad ds^2 = -H^{-2}dt^2 + H^2\delta_{ij}dx^i dx^j$$

$$i, j = 1, 2, 3$$

$$(\text{II.14}) \qquad \delta^{ij}\partial_i\partial_j H = 0 \qquad \text{where } \delta^{ij} \text{ is the Kronecker delta}$$

$$(\text{II.15}) \qquad A_t = H^{-1}$$

Completely in terms of a harmonic function H. [A harmonic function must satisfy the Laplace equation: $\nabla^2 F = 0$ or in this case $\nabla^2 H = 0$]. Such solutions are known as extremal ($Q = M$). They represent stable black holes.

III. MY PARTICULAR PROBLEM

Supergravity theories arise from string theory as low energy limits. They are classical theories of gravity and other types of fields. Solutions to the nonlinear Einstein and field equations of supergravity are necessarily solutions of string theory as well. Many such solutions are known as *p*-branes; extended objects spanning *p* spatial dimensions. A 0-brane is a point-like object while a 1-brane is string-like and so on. The hunt for and categorization of such solutions has been particularly popular in the literature in recent years. Keep in mind that branes are generalizations of charged black holes [as described in the previous section], where a charged black hole like the Reissner-Nordstrøm black hole is a 0-brane like object charged electrically. So if we define a black hole to be a 0-brane then a string is a 1-brane and what we are working with is a 2-brane.

We are interested in a solution in type IIA supergravity that corresponds to two intersecting 2-branes coupled to a 3-form gauge potential, and a scalar field traditionally known as the dilaton. We derive the equations of motion of the theory and construct an ansatz to the solution. The purpose of this study is to classify as many solutions as possible to string theory because these solutions may lead to understanding some cosmological applications of branes. Brane solutions may also provide us clues to understanding nonlinear theories in general. Other, previously known, intersecting brane solutions[6] have provided the groundwork for further understanding the full structure of string theory and how it relates to other theories in lower dimensions. An example of a metric for a single 2-brane in D=10 spacetime looks like:

$$(\text{III.1}) \qquad ds_{10}^2 = f^{-2/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + f^{1/3}\delta_{\mu\nu}dx^\mu dx^\nu$$

where $\mu = 3,\ldots,9$, $\nu = 3,\ldots,9$

$$(\text{III}.2) \qquad \delta^{\mu\nu}\partial_{\mu}\partial_{\nu}f = 0$$

$$(\text{III}.3) \qquad A_{t12} = \pm f^{-1}$$

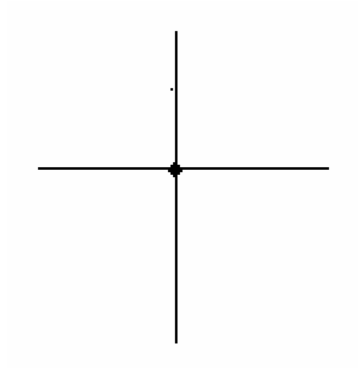
where f is a harmonic function and A_{t12} is the gauge field.
(Note the similarity with the Reissner-Nordstrøm solution)

To get a better understanding of how these branes can intersect over just one point I am including a few tables to show how they intersect along with an example of how two wires and two planes intersect for a reference point (on page 18).

[6] Such as Nucl. Phys. **B** 478:544-560, 1996 and Phys. Rev. **D** 63:064003, 2001

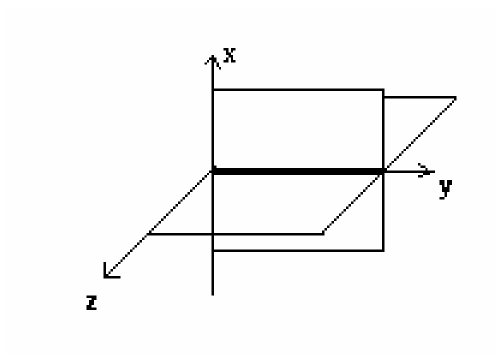
	4-Dimensions			
	0	1	2	3
Wire 1	x	x		
Wire 2	x		x	

"These two wires run in two different spatial dimensions intersect over a point in the time dimension."



	4-Dimensions			
	0	1	2	3
Plane 1	x	x	x	
Plane 2	x		x	x

"These two planes traverse two different sets of coordinates and they intersect over a line because of the intersection in one dimension of space and one of time."



	10-Dimensions									
	0	1	2	3	4	5	6	7	8	9
2-brane 1	x	x	x							
2-brane 2	x			x	x					

"Now we see that our two 2-branes intersect over a point in the time dimension because they are in two separate space dimensions that do not intersect unlike the planes that were shown above."

IV. Calculations

The problem that we are looking at uses this as our action:

$$(\text{IV.1}) \qquad S_{IIA} = \int d^{10}x\sqrt{-G}e^{\Phi}(R - \frac{1}{2}F^{MNLP}F_{MNLP})$$

$$(\text{IV.2}) \qquad F_{MNPL} = \partial_{[M}A_{NLP]}$$

$$M, N, L, P = 0,...,9$$

Φ is the dilaton

G is the determinant of the metric

The square brackets represent the antisymmetrization of the indices.

Using the ansatz,

$$(\text{IV.3}) \qquad ds_{10}^2 = -(f_1 f_2)^{-2A}dt^2 + (f_1^{-2A}f_2^{2B})(dx_1^2 + dx_2^2) + (f_1^{2B}f_2^{-2A})(dx_3^2 + dx_4^2) + (f_1 f_2)^{2B}\delta_{\mu\nu}dx^{\mu}dx^{\nu}$$

$$\mu = 5,...,9, \nu = 5,...,9$$

We constructed this ansatz from the premises given on pg. 17. The second and third terms of this equation describe the two 2-branes, while the last term is the space that is orthogonal to both. The functions f_1 & f_2 are unknown functions which we will show satisfy the Laplace equation.

And our dilaton (an independent constant scalar field) looks like,

$$(\text{IV.4}) \qquad e^{\Phi} = (-f_1 f_2)^E$$

The dilaton is present in this calculation because it is in the general action of any Type IIA String Theory.

Here are our gauge potential and the following ansatz, which is based on the single 2-brane in the previous pages,

$$(\text{IV.5}) \qquad A_{t12} = -Cf_1^{-D}$$
$$(\text{IV.6}) \qquad A_{t34} = -Cf_2^{-D}$$

$$(\text{IV.7}) \qquad F_{\mu t12} = (\partial_\mu A_{t12}) \qquad\qquad (\text{IV.9}) \qquad F_{\mu t12} = -CDf_1^{-D-1}(\partial_\mu f_1)$$
$$(\text{IV.8}) \qquad F_{\mu t34} = (\partial_\mu A_{t34}) \qquad\qquad (\text{IV.10}) \qquad F_{\mu t34} = -CDf_2^{-D-1}(\partial_\mu f_2)$$

note that t is time

In this problem we are looking for the constants A, B, C, D, and E from our derviations as follows:

Deriving our equations of motion from varying our action,

$$(\text{IV.11}) \qquad L = -\frac{1}{2}e^{\Phi}F_{MNLP}F^{MNLP}\sqrt{-G}$$

as we vary the Lagrange equation (IV.11) we find,

$$(\text{IV.12}) \qquad \frac{\partial L}{\partial A} = 0$$

$$(\text{IV.13}) \qquad \frac{\partial L}{\partial(\partial_M A_{NLP})} = -\frac{1}{2}\sqrt{-G}e^{\Phi}48F^{MNLP}$$

that leads us to the following equations:

$$(\text{IV.14}) \qquad \partial_M\left[\frac{\partial L}{\partial(\partial_M A_{NLP})}\right] = -\frac{1}{2}\partial_M\left[\sqrt{-G}e^{\Phi}48F^{MNLP}\right] = 0$$

$$(\text{IV.15}) \qquad \frac{\partial L}{\partial(\partial_M A_{NLP})} = -\frac{1}{2}\sqrt{-G}e^{\Phi}48F^{MNLP}$$

$$(\text{IV.16}) \qquad \partial_\mu\left(\sqrt{-G}e^{\Phi}F^{MNLP}\right) = 0$$

from (IV.16) we get our case to be,

$$(\text{IV.17}) \qquad \partial_\mu\left(\sqrt{-G}e^{\Phi}F^{\mu t12}\right)=\partial_\mu\left(\sqrt{-G}e^{\Phi}F^{\mu t34}\right)=0$$

where,

$$(\text{IV.18}) \qquad F^{\mu t12}=g^{\mu\nu}g^{tt}g^{11}g^{22}F_{\mu t12}$$

$$(\text{IV.19}) \qquad F^{\mu t34}=g^{\mu\nu}g^{tt}g^{33}g^{44}F_{\mu t34}$$

$$(\text{IV.20}) \qquad g^{tt}=-\left(f_1f_2\right)^{2A}$$

$$(\text{IV.21}) \qquad g_{tt}=-\left(f_1f_2\right)^{-2A}$$

$$(\text{IV.22}) \qquad g^{\mu\nu}=\delta^{\mu\nu}\left(f_1f_2\right)^{-2B}$$

$$(\text{IV.23}) \qquad g_{\mu\nu}=\delta_{\mu\nu}\left(f_1f_2\right)^{2B} \ \ \textit{... etc.}$$

$$\mu,\nu=5,...,9$$

To find G, where G is the determinant of our metric g .

$$g=\begin{bmatrix}
-(f_1f_2)^{-2A} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & f_1^{-2A}f_2^{2B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & f_1^{-2A}f_2^{2B} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & f_1^{2B}f_2^{-2A} & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & f_1^{2B}f_2^{-2A} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & (f_1f_2)^{2B} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & (f_1f_2)^{2B} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (f_1f_2)^{2B} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (f_1f_2)^{2B} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (f_1f_2)^{2B}
\end{bmatrix}$$

So for the first part of this calculation we are looking to satisfy:

$$(\text{IV.24}) \qquad \partial_\mu\left(\sqrt{-G}e^{\Phi}F^{\mu t12}\right)=\partial_\mu\left(\sqrt{-G}e^{\Phi}F^{\mu t34}\right)=0$$

First we need to look at our ansatz written out explicitly:

$$(\mathrm{IV.25}) \qquad F^{\mu t12} = -CD\delta^{\mu\nu} f_1^{-2B+6A-D-1} f_2^{-6B+2A}(\partial_\nu f_1)$$

$$(\mathrm{IV.26}) \qquad F^{\mu t34} = -CD\delta^{\mu\nu} f_1^{-6B+2A} f_2^{-2B+6A-D-1}(\partial_\nu f_2)$$

substituting (IV.25 & IV.26) into:

$$(\mathrm{IV.27}) \qquad \partial_\mu\left(\sqrt{-G}(-f_1f_2)^E\left(-CD\delta^{\mu\nu} f_1^{-2B+6A-D-1} f_2^{-6B+2A}(\partial_\nu f_1)\right)\right) = \partial_\mu\left(\sqrt{-G}(-f_1f_2)^E\left(-CD\delta^{\mu\nu} f_1^{-6B+2A} f_2^{-2B+6A-D-1}(\partial_\nu f_2)\right)\right) = 0$$

which eventually reduces down to, where we have taken out the harmonic functions because they satisfy equation (IV.29):

$$(\mathrm{IV.28}) \qquad \begin{bmatrix} -CD\begin{bmatrix} (3A+5B+E-D-1) f_1^{3A+5B+E-D-2}\left(\partial_\mu f_1\right) f_2^{-A+B+E}\left(\partial^\mu f_1\right) + \\ (-A+B+E) f_2^{-A+B+E-1}\left(\partial_\mu f_2\right) f_1^{3A+5B+E-D-1}\left(\partial^\mu f_1\right) \end{bmatrix} - \\ CD\begin{bmatrix} (-A+B+E) f_1^{-A+B+E-1}\left(\partial_\mu f_1\right) f_2^{3A+5B+E-D-1}\left(\partial^\mu f_2\right) + \\ (3A+5B+E-D-1) f_2^{3A+5B+E-D-2}\left(\partial_\mu f_2\right) f_1^{-A+B+E}\left(\partial^\mu f_2\right) \end{bmatrix} \end{bmatrix} = 0$$

$$(\mathrm{IV.29}) \qquad \delta^{\mu\nu}\left(\partial_\mu\partial_\nu f_1\right) = \delta^{\mu\nu}\left(\partial_\mu\partial_\nu f_2\right) = 0$$

The fact that f_1 & f_2 are harmonic seems to indicate that the solution preserves at least some part of supersymmetry. The coefficients must then vanish:

$$\left.\begin{array}{ll} (\mathrm{IV.29}) & 3A+5B+E-D-1=0 \\ (\mathrm{IV.30}) & -A+B+E=0 \end{array}\right\} \quad \text{from } \nabla^2 F = 0$$

From here we cannot solve for all 5 unknowns we have, so we turn to the Einstein equation to give us the remaining equations we need to find out what the unknowns we have actually are.

$$(\text{IV.31}) \qquad R_{MN} - \frac{1}{2} g_{MN} R = 8\pi T_{MN}$$

where,

R_{MN} is the Ricci tensor

R is the Ricci scalar

T_{MN} is the energy-momentum tensor

Note: the Ricci scalar comes from

$$(\text{IV.31a}) \qquad R = g^{\mu\nu} R_{\mu\nu} = \left(\partial_\alpha \Gamma^\alpha_{\mu\nu}\right) - \left(\partial_\nu \Gamma^\alpha_{\mu\alpha}\right) + \Gamma^\beta_{\mu\nu}\Gamma^\alpha_{\beta\alpha} - \Gamma^\beta_{\mu\alpha}\Gamma^\alpha_{\beta\nu}$$

where the Christoffel symbols are:

$$(\text{IV.31b}) \qquad \Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}\left[\left(\partial_\mu g_{\beta\nu}\right) + \left(\partial_\nu g_{\beta\mu}\right) - \left(\partial_\beta g_{\mu\nu}\right)\right]$$

Let's contract both sides of (IV.31) by the inverse metric to give us,

$$(\text{IV.32}) \qquad g^{MN} R_{MN} - \frac{1}{2} g^{MN} g_{MN} R = 8\pi g^{MN} T_{MN} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

using the definitions (IV.33 & IV.34):

$$(\text{IV.33}) \qquad g^{MN} R_{MN} = R$$

$$(\text{IV.34}) \qquad g^{MN} g_{MN} = \delta^M_M = 10$$

will lead us to,

$$(\text{IV}.35) \qquad R - 5R = 8\pi g^{MN} T_{MN}$$

$$(\text{IV}.36) \qquad -4R = 8\pi g^{MN} T_{MN}$$

and we know that (IV.37) is true:

$$(\text{IV}.37) \qquad T_{MN} = \frac{1}{8\pi} \frac{1}{\sqrt{-g}} \frac{\partial L_{matter}}{\partial g^{MN}}$$

where we know the matter part of the Lagrange equation to be,

$$(\text{IV}.38) \qquad L_{matter} = -\frac{1}{2}\sqrt{-g}\, e^{\Phi} F_{MNLP} F^{MNLP}$$

you'll recall from pg. 20,

$$(\text{IV}.39) \qquad F^{MNLP} = g^{MA} g^{NB} g^{LC} g^{PD} F_{MNLP}$$

and also we know,

$$(\text{IV}.40) \qquad L_{matter} = -\frac{1}{2}\sqrt{-g}\, e^{\Phi} F_{MNLP} F_{ABCD}\, g^{MA} g^{NB} g^{LC} g^{PD}$$

$$(\text{IV}.41) \qquad T_{EF} = \frac{1}{8\pi} \frac{1}{\sqrt{-g}} \frac{\partial L_{matter}}{\partial g^{EF}}$$

$$(\text{IV}.42) \qquad \frac{\partial L_{matter}}{\partial g^{EF}} = -\frac{1}{2} \frac{\partial \sqrt{-g}}{\partial g^{EF}} e^{\Phi} F^2 - \frac{1}{2}\sqrt{-g}\, e^{\Phi} F_{MNLP} F_{ABCD} \left(g^{MA} g^{NB} g^{LC} \delta_E^P \delta_F^D + 3 \textit{ more terms...} \right)$$

$$(\text{IV}.43) \qquad \frac{\partial L_{matter}}{\partial g^{EF}} = -\frac{1}{2}\sqrt{-g}\, g_{EF} e^{\Phi} F^2 - \frac{1}{2}^2 4\sqrt{-g}\, e^{\Phi} F_{MNLE} F^{MNL}{}_F$$

$$(\text{IV}.44) \qquad T_{EF} = -\frac{1}{8\pi}\left(\frac{1}{4} g_{EF} e^{\Phi} F_{MNLP} F^{MNLP} - 2e^{\Phi} F_{MNLE} F^{MNL}{}_F\right)$$

$$(\text{IV}.45) \qquad G_{EF} = e^{\Phi}\left(R_{EF} - \frac{1}{2} g_{EF} R\right) = 8\pi T_{EF}$$

$$(\text{IV}.46) \qquad e^{\Phi}\left(R_{EF} - \frac{1}{2} g_{EF} R\right) = -\frac{1}{4} g_{EF} e^{\Phi} F_{MNLP} F^{MNLP} + 2e^{\Phi} F_{MNLE} F^{MNL}{}_F$$

therefore,

$$(\text{IV}.47) \qquad G_{EF} = e^{\Phi}\left(R_{EF} - \frac{1}{2} g_{EF} R\right) = 8\pi T_{EF}$$

$$(\text{IV}.48) \qquad g^{EF}\left(R_{EF} - \frac{1}{2} g_{EF} R = -\frac{1}{4} g_{EF} F_{MNLP} F^{MNLP} + 2F_{MNLE} F^{MNL}{}_F\right)$$

which gives us,

$$(\text{IV}.49) \qquad R - \frac{1}{2} 10R = -\frac{1}{4} 10F^2 + 2F^2$$

$$(\text{IV}.50) \qquad -4R = -\frac{1}{2} F^2$$

$$(\text{IV}.51) \qquad R = \frac{1}{8} F^2$$

that leads us to,

$$(\text{IV}.52) \qquad R_{EF} = -\frac{3}{16} g_{EF} F_{MNLP} F^{MNLP} + 2F_{MNLE} F^{MNL}{}_F$$

The indices have changed names but that is simply because for our purposes the indices that relate to T and g are dummy indices that are simply being used to sum over. As we keep going with this solution we'll find:

$$(\text{IV.53}) \qquad 2f_1^{-2(1+B)}f_2^{-2(1+B)}\begin{bmatrix}3\left(A+2A^2-5AB+B(-2+5B)\right)f_1^2\left(\partial_\mu f_2^2\right)+3f_2^2\left(A+2A^2-5AB+B(-2+5B)\partial_\mu f_1^2\right)- \\ (A-2B)f_1\partial_\mu^2 f_1+(A-2B)f_1 f_2\left(2(5A-7B)\partial_\mu f_1\partial_\mu f_2-3f_1\partial_\mu^2 f_1\right)\end{bmatrix}=$$

$$-\frac{4!}{8}CD\left(f_1^{-2B+6A-2D-2}f_2^{6B+2A}\partial_\mu f_1\delta^{\mu\nu}\partial_\nu f_1+f_1^{2A-6B}f_2^{6A-2B-2-2D}\delta^{\mu\nu}\partial_\mu f_2\partial_\nu f_2\right)$$

which leads us to,

$$(\text{IV.54}) \qquad -2f_1^{-2(1+B)}f_2^{-2(1+B)}\left[3\left(A+2A^2-5AB+B(-2+5B)\right)f_1^2\left(\partial_\mu f_2\partial_\nu f_2\right)\right]=\left(f_1^{2A-6B}f_2^{6A-2B-2-2D}\delta^{\mu\nu}\partial_\mu f_2\partial_\nu f_2\right)$$

$$\left.\begin{aligned}&(\text{IV.55}) \qquad -2B=2A-6B \\ &(\text{IV.56}) \qquad -2B-2=6A-2B-2-2D\end{aligned}\right\} \text{exponents}$$

$$\left.\begin{aligned}&(\text{IV.57}) \qquad -2\left[3\left(A+2A^2-5AB+B(-2+5B)\right)\right]=\frac{4!}{8}CD \\ &(\text{IV.58}) \qquad 2(5A-7B)(A-2B)=0\end{aligned}\right\} \text{coefficients}$$

equations IV.55 - 58 from $\qquad R=\frac{1}{8}F^2$

From these equations we will see that we do not need (IV.58) because one of the solutions it gives does not satisfy our other equations and therefore cannot be true and the other solution is already known from (IV.55). Also note that:

$\delta^{\mu\nu}\left(\partial_\mu\partial_\nu f_1\right)=\delta^{\mu\nu}\left(\partial_\mu\partial_\nu f_2\right)=0$ and are confirmed in the Einstein equation.

Solving for these equations gives us the following solution:

Using (IV.55)

$$-2B = 2A - 6B$$
$$4B = 2A$$
$$2B = A$$

Using (IV.56)

$$-2B - 2 = 6A - 2B - 2 - 2D$$
$$0 = 6A - 2D$$
$$3A = D$$

Using (IV.30)

$$-A + B + E = 0$$
$$B - 2B + E = 0$$
$$-B + E = 0$$
$$B = E$$

Using (IV.29)

$$3A + 5B - D - 1 + E = 0$$
$$D + 5B - D - 1 + E = 0$$
$$6E - 1 = 0$$

giving us,

$$(\text{IV.59}) \qquad E = \frac{1}{6}$$
$$(\text{IV.60}) \qquad B = \frac{1}{6}$$
$$(\text{IV.61}) \qquad A = \frac{1}{3}$$
$$(\text{IV.62}) \qquad D = 1$$

Then our final equation (IV.57) gives us:

$$(\text{IV.63}) \qquad -B^2\left(A + 2A^2 - 5AB - 2B + 5B^2\right) = 3C^2D^2$$
$$(\text{IV.64}) \qquad -2\left(\frac{1}{3} + \frac{2}{9} - \frac{5}{18} - \frac{2}{6} + \frac{5}{36}\right) = C^2$$
$$(\text{IV.65}) \qquad -\frac{1}{6} = C^2$$
$$(\text{IV.66}) \qquad C = \pm\frac{i}{\sqrt{6}}$$

Note: the apperance of i is not unusual, as it is conventional to have an "i" in the equations of motion that would have cancelled this "i" but we ignored it and therefore we may remove this "i" by hand as well, giving us,

$$(\text{IV.67}) \qquad C = \pm\frac{1}{\sqrt{6}}$$

The metric for two 2-branes intersecting over a point in type IIA SUGRA, charged under a 3-form gauge field potential and a static dilaton is:

$$(\text{IV.68}) \qquad ds_{10}^2 = -(f_1 f_2)^{-2/3}dt^2 + (f_1^{-2/3}f_2^{1/3})(dx_1^2 + dx_2^2) + (f_1^{1/3}f_2^{-2/3})(dx_3^2 + dx_4^2) + (f_1 f_2)^{1/3}\delta_{\mu\nu}dx^\mu dx^\nu$$
$$\mu = 5,\ldots,9, \nu = 5,\ldots,9$$

$$(\text{IV.69}) \qquad A_{t12} = \pm\frac{1}{\sqrt{6}}f_1^{-1}$$

$$(\text{IV.70}) \qquad A_{t34} = \pm\frac{1}{\sqrt{6}}f_2^{-1}$$

$$(\text{IV.71}) \qquad e^\Phi = (-f_1 f_2)^{1/6}$$

and,

$$(\text{IV.72}) \qquad \delta^{\mu\nu}\left(\partial_\mu \partial_\nu f_j\right) = 0$$
$$j = 1,2$$

V. Conclusions

From all of these calculations we found a solution to type IIA Supergravity which represents two intersecting 2-branes charged under a 3-form gauge field. The proof that we have solved this solution is written in terms of spatial functions that satisfy the Laplace equation, essentially showing that they are harmonic functions, is an indication that the solution is supersymmetric, which we have not explicitly proved but can conclude is true from these calculations. We want to know this because we want to classify as many possible solutions to string theory as are possible. What we have found is a one specific case that may be added to the list of solutions that have already been found so that we may classify these types of solutions (see references for similar solutions). Classifying these solutions may lead to understanding some cosmological applications of branes. Brane solutions may also provide us clues to understanding nonlinear theories in general. And are not limited to string theory they may even lead to applications in particle physics theory as well.

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